

The Long-run Economic Costs of AIDS: Theory and an Application to South Africa*

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Contents

1	Introduction	5
2	Positive Theory	10
2.1	The Model	10
2.1.1	The Household's Behavior	14
2.1.2	Dynamics	17
2.2	Disease, Increasing Inequality and Economic Collapse	19
2.2.1	Short-Run Dynamics	21
2.2.2	Long-Run Dynamics	22
2.3	Pooling as a Social Response	23
2.3.1	The household's behavior under complete pooling	24
2.3.2	The virtues and drawbacks of pooling	26
3	The Policy Problem	27
3.1	Rationale and Instruments	27
3.1.1	Education Policy	28
3.1.2	Health Policy	30
3.1.3	The Government's Budget Constraint	32
3.2	The Efficacy of a Single Policy Instrument	34

3.2.1	Education Policy	34
3.2.2	Health Policy	39
3.3	Double versus Single Targeting	39
3.3.1	Health Spending is a Private Good	40
3.3.2	Health Spending is a Public Good	42
3.4	Multiple Equilibria, Self-Fulfilling Prophecies and the Credibility of Policies	43
4	Fairly Good Policy Programs	46
4.1	Nuclear Families	47
4.1.1	The Second Phase	48
4.1.2	The First Phase	55
4.2	Pooling	59
5	An Application to South Africa	63
5.1	Calibrating the Model	63
5.2	Benchmarks and Policies: Results	73
5.2.1	Benchmark 1: pooling, no AIDS	76
5.2.2	Benchmark 2: pooling, AIDS and no intervention	76
5.2.3	Policy option 1: spending on health under pooling	77
5.2.4	Policy option 2: nuclear families, lump-sum subsidies	78

5.2.5	Policy option 3: nuclear families, school-attendance subsidies . . .	82
5.3	Other Variations and Robustness	83
5.3.1	Mortality	84
5.3.2	The transmission factor z	87
5.3.3	A delayed policy response	89
6	Conclusions	92
7	Appendices	95
7.1	An Example	95
7.1.1	Household decisions	95
7.1.2	Steady states	97
7.2	Proof of Proposition 3	97
7.3	Proof of proposition 5	100
7.4	Proof of Proposition 6	103
7.5	Proof of proposition 7	105
8	Figures	109

1 Introduction

While the costs of AIDS in terms of human suffering and lives lost are undeniably large, estimates of the associated macroeconomic costs have tended to be more modest. For example, studies that focus on Africa – the continent where the epidemic has hit the hardest – calculate the annual loss of GDP to be around one percent (see Table 1). These estimates all stem from a particular view of how the economy functions, namely, where the AIDS-induced increase in mortality reduces the pressure of population on existing land and capital, thereby raising the productivity of labor. Even if there is a decline in savings and investment (from the reallocation of expenditures towards medical care), its impact on GDP growth is dampened by the countervailing effect of increased labor productivity. Consequently, the net effect on the growth rate of per-capita GDP is very modest.

Table 1: The Impact of AIDS on GDP Growth: Selected Studies

Author(s)	Countries	Method	Effect on GDP growth (%)
Arndt and Lewis (2000)	South Africa	CGE simulations	–0.8 to –1.0
Bonnel (2000)	47 Countries	Cross-country regression	–0.7 ^a
Kambou et al. (1992)	Cameroon	CGE simulations	–0.5 to –1.2
Over (1992)	30 Countries	Demographic/ economic modeling	–0.3 to –0.6
Sackey and Raparla (2000, 2001a,b)	Lesotho, Namibia, Swaziland	Demographic/ economic modeling	–0.8 to –1.5 ^a

^a GDP per capita

In this paper, we argue that the long-run economic costs of AIDS are almost certain to be much higher – and possibly devastating. We take a very different view of how the economy functions over the long run, one which emphasizes the importance of human capital and transmission mechanism across generations. The formation of human capital, which should

be thought of as the entire stock of knowledge and abilities (general and specific) embodied in the population, plays a leading role in promoting economic growth. The accumulation of human capital is the force that generates economic growth over the very long run. If, as is highly plausible, the mechanism that drives the process is the transmission of knowledge and abilities from one generation to the next, then a widespread epidemic of AIDS will result in a substantial slowing of economic growth, and may even result in an economic collapse. The implications of this model are particularly relevant to Africa, as it is the continent with the lowest level of human capital and the highest prevalence of the disease.

The argument establishing how AIDS can severely retard economic growth, even to the point of leading to an economic collapse, is made in three steps. First, AIDS destroys existing human capital in a selective way. It is primarily a disease of young adults. A few years after they become infected, it reduces their productivity by making them sick and weak, and then it kills them in their prime, thereby destroying the human capital progressively built up in them through child-rearing, formal education, and learning on the job.

Second, AIDS weakens or even wrecks the mechanisms that generate human capital formation. In the household, the quality of child-rearing depends heavily on the parents' human capital, as broadly defined above. If one or, worse, both parents die while their offspring are still children, the transmission of knowledge and potential productive capacity across the two generations will be weakened. At the same time, the loss of income due to disability and early death reduces the lifetime resources available to the family, which may well result in the children spending much less time (if any at all) at school. The outcome can be quite pathological. Finally, the chance that the children themselves will contract the disease in adulthood makes investment in their education less attractive, even when both parents themselves remain uninfected. The weakening of these transmission processes is insidious; for its effects are felt only over the longer run, as the poor education of children today translates into low productivity of adults a generation hence.

Third, as the children of AIDS victims become adults with little education and limited knowl-

edge received from their parents, they are in turn less able to raise their own children and to invest in their education. A vicious cycle ensues. If nothing is done, the outbreak of the disease will eventually precipitate a collapse of economic productivity. In the early phases of the epidemic, the damage may appear to be slight. But as the transmission of capacities and potential from one generation to the next is progressively weakened and the failure to accumulate human capital becomes more pronounced, the economy will begin to slow down, with the growing threat of a collapse to follow.

This is the essence of the argument. It has two particularly important implications for economic policy. The first is fiscal in nature. By killing off mainly young adults, AIDS also seriously weakens the tax base, and so reduces the resources available to meet the demands for public expenditures, including those aimed at accumulating human capital, such as education and health services not related to AIDS. Thus, for any given level of fiscal effort, the deleterious effects of the disease on economic growth over the longer run are intensified through this channel. As a result, the state's finances will come under increasing pressure. Slower growth of the economy means slower growth of the tax base, an effect that will be reinforced if there are growing expenditures on treating the sick and caring for orphans.

The other effect is to exacerbate inequality. If the children left orphaned are not given the care and education enjoyed by those whose parents remain uninfected, the weakening of the inter-generational transmission mechanism will express itself in increasing inequality among the next generation of adults and the families they form. Social customs of adoption and fostering, however well-established, may not be able to cope with the scale of the problem generated by a sharp increase in adult mortality, thereby shifting the onus onto the government. As just argued, however, the government itself is likely to experience increasing fiscal difficulties, and so lack the resources to assume this additional burden in full.

In addition to the contributions on the macroeconomic effects of AIDS discussed above, the present paper is related to other strands of the literature. It is motivated, in part, by the empirical observation that good health has a positive and statistically significant effect

on aggregate output (Barro and Sala-I-Martin, 1995; Bloom and Canning, 2000; Bloom, Canning and Sevilla, 2001). The recent report by the Commission on Macroeconomics and Health (WHO, 2001) has also stressed that widespread diseases are a formidable barrier to economic growth.

In order to analyze the long-term effects of AIDS, however, some of the specific features of the relationship between health and economic growth must be treated in detail. In particular, there must be a link between the course of the epidemic and economic growth, in the form of feedbacks from premature mortality to education, the formation of human capital and output. For this purpose, we extend the overlapping generations (OLG) model of Bell and Gersbach (2002), which analyzes the nexus of child labor, education and growth, in order to deal with disease-ridden environments, in which some existing level of premature adult mortality is increased by the outbreak of an epidemic and can be mitigated by spending on measures designed to combat it. Parents have preferences over current consumption and the level of human capital attained by their children, making due allowances for early mortality in adulthood. The decision about how much to invest in education is influenced by premature adult mortality in two ways: first, the family's lifetime income depends on the adults' health status, and second, the expected pay-off depends on the level of premature mortality among children themselves when they attain adulthood. The outbreak of AIDS leads to an increase in such mortality, and if the prevalence of the disease becomes sufficiently high, there may be progressive collapse of human capital and productivity.

The policy problem, therefore, is to avoid such a collapse. The instruments available for this purpose are (i) spending on measures to contain the disease and treat the infected, (ii) aiding orphans, in the form of income-support or subsidies contingent on school attendance, and (iii) taxes to finance the expenditure program. The central policy problem is to find the right balance among these interventions in order to ensure economic growth over the long run without excessive inequality.

The plan of the paper is as follows. In the first two parts of the paper, we approach con-

ceptually the questions of how AIDS impinges on the economy and how its effects can be combatted by suitable policies. The basic model is set out in section 2.1, with special attention paid to the effects of premature adult mortality on (nuclear) family structure and the schooling of children. The dynamics of the system under an exogenous mortality profile, corresponding to some given disease environment, are analyzed in section 2.2. Pooling, where members of the extended family take in orphans, as an alternative form of organization that has both advantages and drawbacks in such a setting, is analyzed in section 2.3.

This sets the stage for the analysis of the policy problem in section 3, where the outbreak of the AIDS epidemic is modeled as an adverse shock to an existing profile of premature adult mortality. Interventions in the spheres of health and education are examined separately in section 3.2, and jointly in section 3.3. Finding the right balance between these two sets of measures is the central policy problem, and the results in these sections attempt to illuminate how the balance should be struck. The possibilities of multiple equilibria and self-fulfilling prophecies, together with the associated credibility of public policy, are pursued in section 3.4. An extensive treatment of what can be called ‘fairly good’ policy programs is the subject of section 4, in which the aim is to develop simpler alternatives than those requiring the computation of a full optimum.

In the third part of the paper we apply the model to South Africa. The choice of South Africa as a test-bed is a natural one on several grounds. First, the very nature of the model demands that the available economic and demographic series be long and fairly reliable if there is to be a solid base for calibration. Second, South Africa is a middle-income country that has experienced substantial growth over much of the past half century. A collapse of the kind analyzed in the theoretical sections of this paper, were it to occur, would therefore mean that there is a long way to fall. Third, the epidemic has progressed rapidly in South Africa, from a prevalence rate among the population aged 15 to 49 of about one per cent in 1990 to just over 20 per cent a decade later (UNAIDS, 2002). The demographic writing is already on the wall (see especially Dorrington *et al.*, 2001).

The calibration of the model is described in section 5.1. On this basis, the model is used, in section 5.2, to generate the trajectories of the main variables in a variety of settings. The government's task is to choose, within certain restrictions, a sequence of tax schedules, in order to yield the resources to finance a sequence of expenditures on, respectively, measures to combat the epidemic and the support of needy children so as to induce their education. There are two reference cases, one of which is a counterfactual without the epidemic, and several policy variations, depending on the instruments actually available to the government. A whole array of sensitivity tests is employed in section 5.3 to assess the robustness of the findings in section 5.2.

The last section is devoted to an assessment of the overall results and the most fruitful directions of future research.

2 Positive Theory

2.1 The Model

We extend the OLG-model of Bell and Gersbach (2002) by introducing premature mortality among adults. There are two periods of life, childhood and adulthood, whereby the course of adulthood runs as follows. On becoming adults, individuals immediately form families and have their children. When the children are very young, they can neither work nor attend school. Since the only form of investment is education, the family's full income is wholly consumed in this phase. Only after this phase is over do the adults learn whether they will die prematurely, and so leave their children as half- or full orphans. Early in each generation of adults, therefore, all nuclear families are sorted into one of the following four categories:

1. both parents survive into old age,
2. the father dies prematurely,

3. the mother dies prematurely,
4. both parents die prematurely.

These states are denoted by $s_t \in S_t := \{1, 2, 3, 4\}$. The probability that a family formed at the start of period t lands in category s_t is denoted by $\pi_t(s_t)$. The population is assumed to be large enough that this is also the fraction of all families in that state after all premature adult deaths have occurred. An important consequence of such mortality is that it results in heterogeneity among each cohort of families. Once their states have been revealed, families make their decisions accordingly, as will be described below.

We turn to the formation of human capital. Consider a family at the start of period t . Let λ_t^f and λ_t^m denote, respectively, the father's and mother's endowments of human capital, and let $\Lambda_t(s_t)$ denote their total human capital when the family is revealed to be in state s_t . Then,

$$\Lambda_t(1) = \lambda_t^f + \lambda_t^m, \Lambda_t(2) = \lambda_t^m, \Lambda_t(3) = \lambda_t^f, \Lambda_t(4) = 0. \quad (1)$$

An additional source of heterogeneity is ruled out in advance:

Assumption 1. There is assortative mating: $\lambda_t^f = \lambda_t^m \forall t$.¹

Hence, (1) specializes to

$$\Lambda_t(1) = 2\lambda_t, \Lambda_t(2) = \Lambda_t(3) = \lambda_t, \Lambda_t(4) = 0, \quad (2)$$

where the superscripts f and m may be dropped without introducing ambiguity.

Human capital is assumed to be formed by a process of child-rearing combined with formal education in the following way. In the course of rearing their children, parents give them

¹This assumption is solely made for simplicity of the exposition of our main arguments.

a certain capacity to build human capital for adulthood, a capacity which is itself increasing in the parents' own human capital. This gift will be of little use, however, unless it is complemented by at least some formal education, in the course of which the basic skills of reading, writing and calculating can be learned. Let the proportion of childhood devoted to education be denoted by $e_t \in [0, 1]$, the residual being allocated to work, and for simplicity, let all the children in a family be treated in the same way. Expressed formally, the human capital attained by each of the children on reaching adulthood is assumed to be given by

$$\lambda_{t+1} = \begin{cases} z(s_t)f(e_t)\Lambda_t(s_t) + 1, & s_t = 1, 2, 3 \\ \xi & s_t = 4 \end{cases} \quad (3)$$

Beginning with the upper branch of (3), the term $z_t(s_t)$ represents the strength with which capacity is transmitted across generations. It is plausible that the father's and mother's contributions to this process are not perfect substitutes, in which case, $2z(1) > \max[z(2), z(3)]$ and $z(2)$ may not be equal to $z(3)$. For simplicity, however, we introduce

Assumption 2. $z(2) = z(3) \geq z(1) \geq z(2)/2 = z(3)/2$.

$z(1) = z(2) = z(3)$ holds when the parents are perfect complements and $2z(1) = z(2) = z(3)$ when they are perfect substitutes. Assumptions 1 and 2 allow the upper branch of (3) to be rewritten as

$$\lambda_{t+1} = (3 - s_t)z(s_t)f(e_t)\lambda_t + 1, \quad s_t = 1, 2 \quad (4)$$

both types of single-parent families being identical in this respect. The function $f(\cdot)$ may be thought of as representing the educational technology – translating time spent on education into learning.

Assumption 3. $f(\cdot)$ is a continuous, strictly increasing and differentiable function on $[0, 1]$, with $f(0) = 0$.

Observe that assumption 3 implies that children who do not attend school at all attain, as adults, only some basic level of human capital, which has been normalized to unity. A whole society of such adults will be said to be in a state of backwardness.

According to the lower branch of (3), there is a miserable outcome for full orphans who do not enjoy the good fortune to be adopted or placed in (good) institutional care. Deprived of love and care, and being left to their own devices, they go through childhood uneducated, to attain human capital $\xi (\leq 1)$ in adulthood.

The next step is to relate human capital to current output, which takes the form of an aggregate consumption good. The following assumption implies that current output will accrue to families as income in proportion to the amounts of labor, measured in efficiency units, that they supply.

Assumption 4. Output is proportional to inputs of labor measured in efficiency units.

A natural normalization is that an adult who possesses human capital in the amount λ_t is endowed with λ_t efficiency units of labor, which he or she supplies completely inelastically.

A child's contribution to the household's income is given as follows: In view of the complementarity between the potential capacity received during rearing and formal education, a child will supply at most one efficiency unit of labor during childhood. Indeed, it is plausible that a child's efficiency will be somewhat lower than the parents', *ceteris paribus*, on grounds of age alone. To reflect these considerations, let a child supply $\gamma(1 - e_t(s_t))$ efficiency units of labor when the child works $1 - e_t(s_t)$ units of time. It is plausible to assume that $\gamma \in (0, \xi)$, i.e. a full-time working child is at most as productive as an adult who happened to be an orphan. A family with n_t children therefore has a total income in state s_t ($s_t = 1, 2, 3$) of

$$y_t(s_t) = \alpha[\Lambda_t(s_t) + n_t(1 - e_t(s_t))\gamma] \quad (5)$$

where the scalar $\alpha(> 0)$ denotes the productivity of human capital, measured in units of output per efficiency unit of labor input.

2.1.1 The Household's Behavior

It is assumed that all allocative decisions lie in the parents' hands, as long as they are alive. We rule out any bequests at death, so that the whole of current income, as given by (5), is consumed. Concerning the allocation of consumption within the family, let the husband and wife enjoy equality as partners, and let each child obtain a fraction $\beta \in (0, 1)$ of an adult's consumption if at least one adult survives. Full orphans ($s_t = 4$) do not attend school, and consume what they produce as child laborers.

From (2), the budget sets of single-mother and single-father households with the same endowments of human capital and the same number of children are identical. In the absence of any taxes or subsidies, the household's budget line may therefore be written as

$$[(3 - s_t) + n_t\beta]c_t(s_t) + \alpha n_t\gamma e_t(s_t) = \alpha[(3 - s_t)\lambda_t + n_t\gamma], \quad s_t = 1, 2 \quad (6)$$

where $c_t(s_t)$ is the level of each adult's consumption. The expression on the LHS represents the costs of consumption and the opportunity costs of the children's schooling. The expression on the RHS is the family's so-called full income² in state $s_t = 1, 2, 3$, whereby assumption 1 ensures that states 2 and 3 are identical where the budget set is concerned. Observe that single-parent households not only have lower levels of full income than their otherwise identical two-parent counterparts, but that they also face a higher relative price of education, defined as $\alpha n_t\gamma / [(3 - s_t) + n_t\beta]$.

In keeping with the rather imperfect state of knowledge about the relationship between AIDS and fertility, we make no attempt to model fertility in a sophisticated way. Let all mortality among children occur in infancy, and suppose that so-called 'replacement fertility' behavior is unhindered by premature adult mortality. Then:

²A household's full income is the scalar product of its endowment vector and the vector of market prices. Here, output is taken as the numéraire.

Assumption 5. Couples have children while they are young until some exogenously fixed number have survived infancy, a target that may vary from period to period.

With n_t thus fixed, the adults wait until the state of the family becomes known, and the survivor(s) then choose some feasible pair $(c_t(s_t), e_t(s_t)) \geq 0$ subject to (6).

Parents are assumed to have preferences over their own current consumption and the human capital attained by their children in adulthood, taking into account the fact that an investment in a child's education will be wholly wasted if that child dies prematurely in adulthood. Let mothers and fathers have identical preferences, and for two-parent households, let there be no 'joint' aspect to the consumption of the pair $(c_t(1), e_t(1))$: each surviving adult derives (expected) utility from the pair so chosen, and these utilities are then added up within the family. In effect, whereas $c_t(1)$ is a private good, the human capital of the children in adulthood is a public good within the marriage. Since all the children attain λ_{t+1} , the only form of uncertainty is that surrounding the number who will not die prematurely as adults, which is denoted by a_{t+1} . Let preferences be separable, with representation

$$EU_t(s_t) = (3 - s_t)[u(c_t(s_t)) + E_t a_{t+1} v(\lambda_{t+1})], \quad s_t = 1, 2 \quad (7)$$

where the contribution $v(\lambda_{t+1})$ counts only when death does not come early, E_t is the expectation operator and $E_t a_{t+1}$ is the expected number of children surviving into old age. The sub-utility functions $u(\cdot)$ and $v(\cdot)$ are assumed to be increasing, continuous, concave and twice-differentiable. Denoting by $\pi_{t+1}(s_{t+1})$ the parents' subjective probability that a child will find itself in state s_{t+1} in period $t + 1$, so that $\sum_{s_{t+1}=1}^4 \pi_{t+1}(s_{t+1}) = 1$, and recalling assumption 1 and that all children are treated identically, we obtain

$$E_t a_{t+1} v(\lambda_{t+1}) = n_t \kappa_{t+1} v(\lambda_{t+1}),$$

where

$$\kappa_{t+1} \equiv [1 + \pi_{t+1}(1) - \pi_{t+1}(4)]/2 \quad (8)$$

and λ_{t+1} is given by the upper branch of (3). Observe that $\kappa_{t+1} = 1$ if and only if there is no premature adult mortality ($\pi_{t+1}(1) = 1$), and that $\kappa_{t+1} < 1$ otherwise. A reduction in κ_{t+1} , therefore, effectively entails a weaker taste for the children's education. By way of illustration, let premature mortality among adults be independently and identically distributed, and denote the probability that an adult will survive to old age by p_t . Then,

$$\pi_t(1) = p_t^2, \pi_t(2) = \pi_t(3) = p_t(1 - p_t), \pi_t(4) = (1 - p_t)^2, \kappa_t = p_t$$

It will be convenient in what follows to rewrite (7) as

$$EU_t(s_t) = (3 - s_t)[u(c_t(s_t)) + n_t \kappa_{t+1} v(z(s_t) f(e_t) \Lambda_t(s_t) + 1)], \quad s_t = 1, 2 \quad (9)$$

A family in state $s_t (= 1, 2, 3)$ in period t solves the following problem:

$$\max_{[c_t(s_t), e_t(s_t)]} EU_t(s_t) \text{ s.t. (6), } c_t(s_t) \geq 0, e_t(s_t) \in [0, 1]. \quad (10)$$

Let $[c_t^0(s_t), e_t^0(s_t)]$ solve problem (10), whose parameters are $(\kappa_{t+1}, \lambda_t, n_t, s_t, \alpha, \beta, \gamma)$. By the envelope theorem, we have

$$\frac{\partial EU_t(s_t)}{\partial \lambda_t} > 0, \frac{\partial EU_t(s_t)}{\partial \kappa_{t+1}} > 0.$$

Since current consumption is maximized by choosing $e_t = 0$, it follows that the parents' altruism towards their children must be sufficiently strong if they are to choose $e_t > 0$.

Assumption 6. Both goods are non-inferior.³

It follows at once that:

$$\frac{\partial e_t^0(s_t)}{\partial \Lambda_t(s_t)} \geq 0 \text{ and } \frac{\partial c_t^0(s_t)}{\partial \Lambda_t(s_t)} \geq 0$$

Inspection of $EU_t(s_t)$ reveals that an increase in κ_{t+1} induces an increase in $e_t^0(s_t)$ if $0 < e_t^0(s_t) < 1$ and preserves $e_t^0(s_t) = 1$; for it increases the weight on $v(\lambda_{t+1})$ relative to

³Note that Λ_t enters both the budget constraint and the utility that adults derive from λ_{t+1} . Therefore, the definition of inferior goods is not the same as the textbook description in this particular set-up.

that on $u(c_t(s_t))$. An increase in κ_{t+1} therefore has the opposite effect on $c_t^0(s_t)$.

The remaining comparative static results concern the effect of family status in the present on investment in, and the accumulation of, human capital. Note that the upper boundaries of the budget sets in the cases $s_t = 2$ and $s_t = 3$ lie strictly inside that associated with $s_t = 1$ and that the price of c_t relative to e_t is lower for $s_t = 2, 3$ than for $s_t = 1$. We then obtain:

Lemma 1

Suppose λ_t is given. Then, under assumptions 1, 2 and 6,

- (i) $e_t^0(1) \geq e_t^0(2) = e_t^0(3)$
- (ii) $\lambda_{t+1}(1) \geq \lambda_{t+1}(2) = \lambda_{t+1}(3)$
- (iii) $\partial e_t^0(s_t) / \partial \kappa_{t+1} > 0$ if $0 < e_t^0(s_t) < 1$.

We now introduce the assumption that altruism is not operative when the adults are uneducated:

Assumption 7. For $\Lambda_t(1) \leq 2$, $e_t^0(1) = 0$.

Part (i) of Lemma 1 then yields $e_t^0(2) = e_t^0(3) = 0$ as a trivial corollary.

2.1.2 Dynamics

Recalling that $e_t^0(s_t)$ is chosen so as to solve problem (10), equation (3) may be written

$$\lambda_{t+1} = \begin{cases} z(s_t) f\left(e_t^0(\Lambda_t(s_t), s_t, \kappa_{t+1})\right) \Lambda_t(s_t) + 1, & s_t = 1, 2, 3 \\ \xi, & s_t = 4 \end{cases} \quad (11)$$

Equation (11) describes a random dynamical system, in the sense that although each child attains λ_{t+1} in adulthood with certainty, he or she can wind up in any of the states $s_{t+1} \in \{1, 2, 3, 4\}$ after reaching adulthood and forming a family. In the absence of premature mor-

tality ($\pi_{t+1}(1) = 1$), =0..infinity the above system has at least two steady states if $z(1)f(1)2\lambda^a + 1 \geq \lambda^a$, where λ^a is the lowest level of an adult's human capital such that a two-parent household chooses full education for the children in such an environment (Bell and Gersbach, 2002).

The typical dynamics in the absence of premature mortality are illustrated in figure 1, where $\Lambda^d (> 2)$ denotes the smallest endowment of the adults' human capital such that they just begin to send their children to school. $\Lambda^a (= 2\lambda^a)$ denotes the corresponding endowment at which children finally enjoy full-time schooling. As depicted, the system has two steady states. First, there is the state of backwardness ($\Lambda = 2$). This stable steady state is a poverty trap, wherein all generations are at the lowest level of human capital. Second, there is an unstable steady state ($\Lambda_t = \Lambda^* \forall t$), in which the parents' human capital is such that they choose a positive level of education for their children that also yields each of the latter $\Lambda^*/2$ in adulthood. To be precise, Λ^* satisfies

$$\frac{\Lambda^*(1)}{2} = z(1)f\left(e_t^0(\Lambda^*(1), 1, 1)\right)\Lambda^*(1) + 1,$$

where $\pi_t(1) = 1$ for all t . Observe that starting from any $\Lambda > \Lambda^*$, unbounded growth is possible if and only if $2z(1)f(1) \geq 1$, and that the growth rate approaches $2z(1)f(1) - 1$ asymptotically.

Matters become more complicated when there is premature adult mortality. First, the values of Λ^d, Λ^* and Λ^a depend both on $s_t \in \{1, 2, 3\}$ and on κ_{t+1} . Second, a separate phase diagram is needed for each pair of states in periods t and $t + 1$. For example, the offspring of a two-parent family in period t all attain the λ_{t+1} corresponding to $\Lambda_t(1)$, but not all their offspring are raised in two-parent families. In principle, therefore, a phase diagram is required for each of the cases in the set $S_t \times S_{t+1}$. The ensuing heterogeneity and its consequences for the system as whole will now be explored in greater detail.

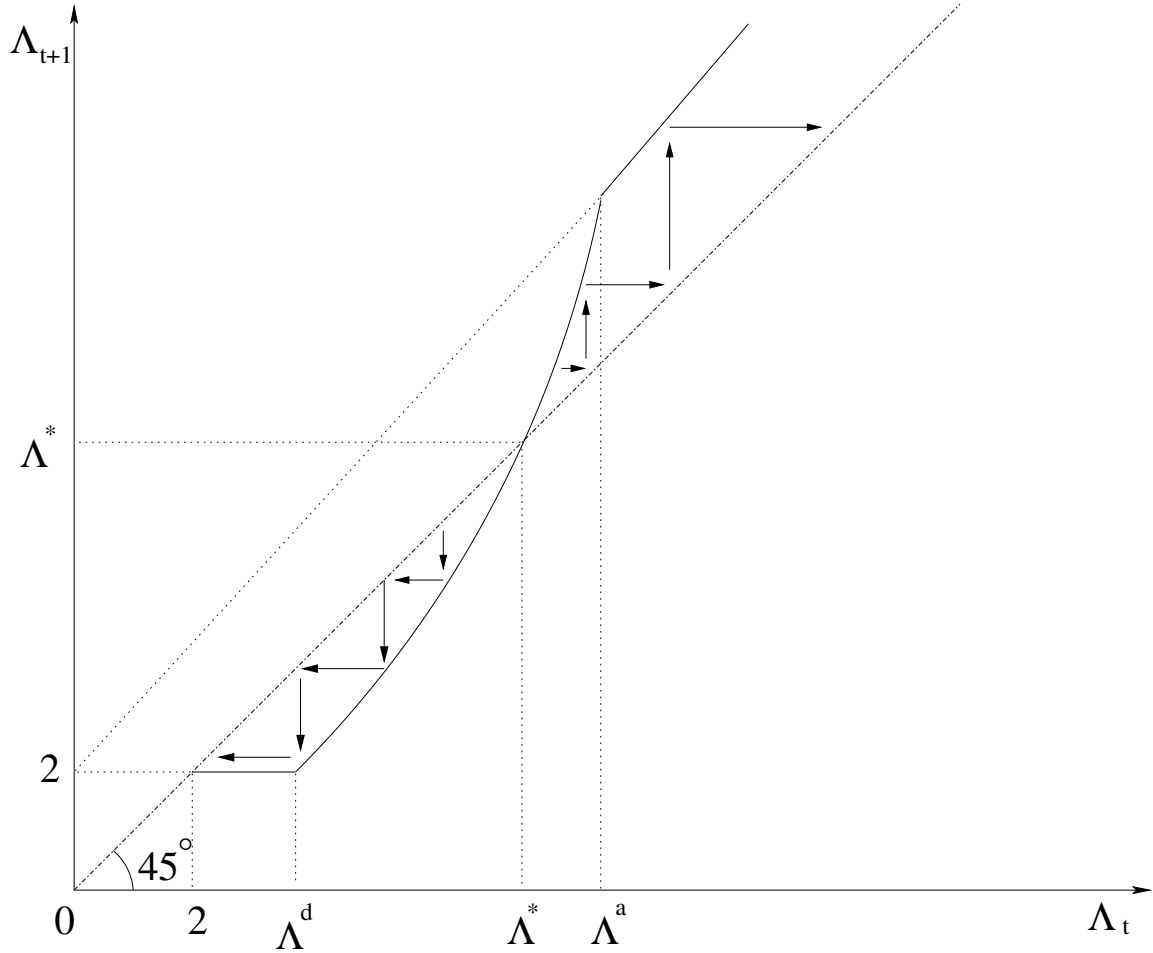


Figure 1: The phase diagram in the absence of premature adult mortality.

2.2 Disease, Increasing Inequality and Economic Collapse

The process by which the onset of a disease like AIDS leads to economic collapse can be described as follows: At the start of period $t = 0$, a society of homogeneous two-parent families, each with adult human capital endowment $2\lambda_0$, is suddenly assailed by some fatal disease. Immediately after their children are born, all adults learn whether they are infected with the disease, and the survivors then choose $(c_0^0(s_0), e_0^0(s_0))$ for $s_0 = 1, 2, 3$. We are interested in the question: how does the outbreak of the disease affect the subsequent development of the society? Children who are left as unsupported orphans ($s_0 = 4$) fall at once into the poverty trap. Assumption 7 also implies that $e_t^0(2\xi, 1) = 0 \forall t$: even if both parents survive but have

been orphans in childhood, they cannot afford to send their children to school. In the absence of support, therefore, all orphans fall into the poverty trap, and their succeeding lineage remains there. In order to discover what happens to the rest, we introduce the critical value function $\lambda^*(s, \kappa_t)$ for $s \in \{1, 2, 3\}$, $n_{t+1} = n_t$, $\forall t$ and $\kappa_t = \kappa$, $\forall t$ defined by:

$$\lambda^*(s, \kappa) = z(s) f(e^0(\Lambda^*(s), s, \kappa)) \Lambda^*(s) + 1 \quad (12)$$

where $\Lambda^*(1) = 2\lambda^*(1)$, $\Lambda^*(2) = \Lambda^*(3) = \lambda^*(2) = \lambda^*(3)$, and κ is a sufficient statistic of premature adult mortality in the steady state. $\lambda^*(s, \kappa)$ is the steady-state human capital associated with a particular state s , that is, in any pair of generations, parent(s) and offspring share the same state.

In order to establish the relationship between $\lambda^*(s, \kappa)$ and $e^0(\Lambda^*(s), s, \kappa)$, we differentiate (12) totally and rearrange terms with respect to κ and obtain:

$$\frac{d\lambda^*}{d\kappa} = \frac{(3-s)z(s)\lambda^* f'(e^0) \frac{\partial e^0}{\partial \kappa}}{\frac{1}{\lambda^*} - (3-s)z(s)f'(e^0) \frac{\partial e^0}{\partial \lambda}} \quad (13)$$

An increase in premature adult mortality increases $\lambda^*(s, \kappa)$, $s = 1, 2, 3$. To be precise, we have

Lemma 2

- (i) $\partial \lambda^*(s, \kappa) / \partial \kappa < 0$, $s = 1, 2, 3$
- (ii) $\lambda^*(1, \kappa) \leq \lambda^*(2, \kappa) = \lambda^*(3, \kappa)$

Proof :

Observe from Figure 1 that the slope of the right hand side of equation (12) is larger than 1 at λ^* . Hence,

$$\frac{\partial}{\partial \lambda} ((3-s)z(s)f(e^0(\Lambda^*(s), s, \kappa))\lambda^*(s)) > 1$$

which implies

$$(3-s)z(s)f'(e^0) \frac{\partial e^0}{\partial \lambda^*} > 1 - (3-s)z(s)f(e^0).$$

By virtue of (12), the right hand side is equal to $\frac{1}{\lambda^*}$ and hence the denominator in (13) is negative. According to the third part of Lemma 1, the numerator in (13) is positive, which

proves the first claim. To establish the second claim, observe that, starting in any period t ,

$$\lambda^*(1, \kappa) = z(1)f(e^0(\Lambda^*, 1, \kappa) \cdot \Lambda^*(1, \kappa) + 1) \geq \lambda_{t+1}(2, \kappa) = z(2)f(e^0(\Lambda^*(2), 2, \kappa) \cdot \Lambda^*(1, \kappa)/2 + 1)$$

by virtue of assumption 2 and the first part of lemma 1. The second claim then follows at once. ■

The first part of lemma 2 implies that an increase in premature adult mortality may cause a group that was earlier enjoying self-sustaining growth to fall into the poverty trap. The second part implies that single-parent families need higher individual levels of human capital than two-parent ones to escape the trap, so that an increase in premature adult mortality also increases the share falling into the poverty trap by increasing the proportion of one-parent families⁴.

2.2.1 Short-Run Dynamics

We now turn to the short-run dynamics following a shock represented by $\kappa_t = \kappa < 1$ for all $t \geq 0$. We denote by P_t the fraction of the population of adults whose human capital is at most unity in period t . Similarly, R_t denotes the fraction of individuals that possess at least $\lambda^*(2, \kappa)$. Note that $P_t + R_t \leq 1$. We obtain the following results:

Lemma 3

Suppose that $\lambda_0 > \lambda^(1, 1)$.*

(i) *If $\lambda_0 \geq \lambda^*(2, \kappa)$, then*

$$P_1 = \pi(4), R_1 = 1 - \pi(4)$$

⁴In appendix 7.1, we provide a detailed analysis of an example, in order to illustrate the most important results from our household model and how the steady state associated with a particular household state $s_t = (1, 2, 3)$ depends on preferences, premature mortality, discounting, the characteristics of child labor and child consumption, and the productivity of human capital. The example also reveals that the existence of a unique, unstable steady state cannot be taken for granted, even when the model does not take its most general form.

(ii) If $\lambda^*(2, \kappa) > \lambda_0 \geq \lambda^*(1, \kappa)$, then

$$P_1 \geq \pi(4), R_1 \leq \pi(1)$$

(iii) If $\lambda^*(1, \kappa) > \lambda_0$, then

$$P_1 \geq \pi(4), R_1 = 0$$

The three claims immediately follow from our preceding discussion. In case (i), families with at least one surviving adult will continue to enjoy self-sustaining growth, although one-parent households will henceforth experience growth at a lower rate if $z(1) > z(2)/2$, even if $e_0^0(2) = 1$. This adverse effect will be reinforced if e^0 falls following the shock. The resulting inequality among families with adults will be propagated into the future, with further differentiation arising both from the transmission factor $z(s)$, and from future differences in $e_t^0(\cdot)$ among them. In case (ii), only families with two adults will continue to experience self-sustaining growth, whereas all the others will descend into the poverty trap. Thereafter, the pattern of progressive differentiation described in case (i) will also take hold here. In case (iii), all families begin to descend into poverty immediately.

2.2.2 Long-Run Dynamics

The preceding discussion yields straightforward implications for long-run dynamics, which are summarized in the next proposition.

Proposition 1

If $\kappa_t < 1$ for all $t \geq 0$, then:

$$(i) P_t \geq P_{t-1} + \pi_{t-1}(4)(1 - P_{t-1}), \quad t \geq 1$$

$$(ii) \lim_{t \rightarrow \infty} P_t = 1$$

Note that part (i) of proposition 1 holds as an equality if $\lambda_0 \geq \lambda^*(2, 1)$. Proposition 1 indicates that the share of uneducated families grows over time until, in the limit, the whole population

is in backwardness. Not only do some adults suffer sickness and early death, but the whole society descends progressively into the poverty trap. This dramatic implication leads one to ask what social arrangements can be made to deal with this danger. One answer is to pool the risks.

2.3 Pooling as a Social Response

The prevailing form of social organization has a potentially important influence on how the economic system copes with premature adult mortality. We can distinguish among three types.

First, there is the family as the nucleus of society (the nuclear family), which is essentially the preceding set-up of our model. Parents are solely responsible for their own children, so that the fortunes of children depend entirely on their natural parents' health status and human capital (or income).

The second involves collective (or pooling) arrangements to some degree. We shall say that partial pooling occurs when a subset of society, be it a region, a city, a tribe, even a very large extended family, pools its resources. It is widely observed that in Africa, for example, orphans are often taken in by, and rotated among, relatives. It is sometimes claimed that the relatives also treat such children as if they were their own; but this goes too far – for instance, Case, Paxson and Abledidinger (2002) show that the schooling of orphans depends heavily on how closely they are related to the adoptive household head. To cover this arrangement, we allow sufficiently large subsets of the society – subintervals in our model – to be pooled, that is to say, all surviving adults in the subset take on joint responsibility for all children in their group. For simplicity, we assume that within each generation, all adults and children in the subset are treated identically. Partial pooling of the society in subintervals is assumed to be on such a scale that aggregate uncertainty outvanishes⁵. Note that partial pooling suffices

⁵It can be shown that, by the same construction as in the general model, aggregate uncertainty cancels out

to diversify completely the idiosyncratic mortality risk, and pooled groups face only the aggregate risk, as summarized by κ . Note also that under partial pooling, children and adults may fare very differently across pooled subsocieties, either because of differences in initial conditions or because public policy favors one subsociety over another.

Thirdly, there is the extreme case of complete pooling, in which all surviving adults in the society take on joint responsibility for all children. It should be remarked that complete pooling does not increase the scope for providing insurance; for, by assumption, partial pooling already suffices to achieve complete insurance against idiosyncratic risks. Complete pooling does impose an equal-treatment constraint on the policies we discuss in the sections that follow. Pooling also introduces the need for a little additional notation: it will be denoted by the family state $s_t = 0$.

2.3.1 The household's behavior under complete pooling

By assumption 5, each couple produces n_t surviving children in period t ; but not all of the adults themselves survive to rear their offspring. Under complete pooling, the children are effectively reared collectively, in the sense that each surviving ‘pair’ of adults raises not n_t , but

$$n_t(0) = \frac{n_t}{\kappa_t} = \frac{2n_t}{1 + \pi_t(1) - \pi_t(4)} \quad (14)$$

children. In effect, the burden of premature adult mortality is borne equally by all surviving members of a generation. The budget line of a representative ‘pair’ is

$$[2 + (n_t/\kappa_t)\beta]c_t(0) + \alpha(n_t/\kappa_t)\gamma e_t(0) = \alpha[2\lambda_t + (n_t/\kappa_t)\gamma], \quad (15)$$

in subintervals that have positive measure.

a comparison of which with (6) reveals that, relative to an otherwise identical two-parent nuclear family, the presence of premature adult mortality implies, first, a lower relative price of current consumption, and second, a lower level of full income, measured in units of an adult's consumption, so long as $\beta > \gamma$. On this score, therefore, a rise in such mortality works to reduce education, relative to the two-parent, nuclear family. Pursuing this point further, it is also seen that by setting κ_t equal to 1 and 1/2, (15) specializes to the cases $s_t = 1$ and $s_t = 2$, respectively, in (6). As we will now see, however, pooling is not necessarily an intermediate case between one- and two-parent nuclear families whenever $\kappa_t \in [1/2, 1]$.

One can think of the pooling arrangement as a representative two-parent family looking after n_t/κ_t children, as opposed to either one or two parents looking after n_t , as analyzed in sections 2.1 and 2.2. In order to bring out this point, the transmission factor under pooling is written as $z(0, \kappa_t)$, where we use the state 0 to denote pooling. If there is no premature adult mortality, pooling is never called into operation, so that $z(0, 1) = z(1)$. If $\kappa_t = 1/2$, the question arises whether two parents can impart a higher potential to each of $2n_t$ children than one parent (of either sex) to n_t ; in keeping with assumption 2, they could hardly do worse. We therefore introduce:

Assumption 8. For any given n_t , $z(0, \kappa_t)$ is a non-decreasing, continuous and differentiable function of κ_t ; it also satisfies $z(0, 1/2) \geq z(2)/2$ and $z(0, 1) = z(1)$.

Hence, the formation of human capital under pooling is given by:

$$\lambda_{t+1} = 2z(0, \kappa_t)f(e_t)\lambda_t + 1 \quad (16)$$

Turning to preferences, let the 'couple' display the same degree of altruism towards natural and adopted children alike, which implies that all children will be treated in the same way. We have

$$EU_t(0) = 2[u(c_t(0)) + n_t(\kappa_{t+1}/\kappa_t)v(2z(0, \kappa_t)f(e_t)\lambda_t + 1)]. \quad (17)$$

Since $\kappa_t < 1$, a comparison of (17) with (9) reveals that there is a greater weight on the childrens' future human capital in the former (pooling) than in the latter (in which the weights

are identical for one- and two-parent families). The assumption that the adults view all children in their care with equal altruism therefore tugs in the opposite direction to that of the price and income effects where investment in education is concerned.

The steady-state value of human capital in the pooling case satisfies

$$\lambda^*(0, \kappa) = 2z(0, \kappa)f(e^0(2\lambda^*(0, \kappa), 0, \kappa)) \cdot \lambda^*(0, \kappa) + 1. \quad (18)$$

Were it not for the force of equal altruism towards all children under pooling, the argument in part (ii) of lemma 2 and assumption 8 would yield the following result: $\lambda^*(1, \kappa) \leq \lambda^*(0, \kappa) \leq \lambda^*(2, \kappa)$ for all $\kappa \in [1/2, 1]$. As it is, an alternative assumption will suffice to ensure that it indeed holds.

Lemma 4

Suppose, by social convention, that all children must be treated identically, but surviving adults value only the future human capital attained by their natural children. Then

$$\lambda^*(1, \kappa) \leq \lambda^*(0, \kappa) \leq \lambda^*(2, \kappa) \quad \text{for all } \kappa \in [1/2, 1].$$

2.3.2 The virtues and drawbacks of pooling

Since pooling is a form of social insurance against premature mortality, it is interesting to ask whether this form of organization is better able to withstand a shock than one based on the nuclear family. The answer turns out to depend on the initial level of human capital.

Proposition 2

Suppose the disease breaks out in period 0, with resulting mortality represented by κ (< 1).

(i) If $\lambda^(0, \kappa) < \lambda_0$, no collapse will occur.*

(ii) If $\lambda^(0, \kappa) > \lambda_0$, the entire group begins an immediate descent into the poverty trap.*

The proof of proposition 2 is straightforward. The outcome in part (i) is in contrast to that in part (ii) of proposition 1. Under pooling, moreover, perfect equality is maintained within each generation. The drawback arises when the change in mortality is so large that the initial level of human capital no longer lies above the critical level in the newly prevailing disease environment. Equality of treatment then pulls everyone down together, whereas in a nuclear family structure with $\lambda^*(1, \kappa) < \lambda_0 < \lambda^*(0, \kappa)$, two-parent families will continue to experience growth. This latter fact plays a very important role when policy interventions are possible, for two-parent families comprise the main tax base in a nuclear family setting.

3 The Policy Problem

3.1 Rationale and Instruments

In the light of section 2, there is a clear and compelling rationale for public intervention, namely, to stave off the economic collapse, with all of its baneful social and human consequences, which an epidemic like AIDS threatens to set in train. In order to draw up a plan of action, it is important to identify the four main reasons why policy intervention is desirable in settings of the present kind. The first and main case for intervention rests on the externalities that arise when the improvements in all future generations' welfare that would stem from a better education of today's children and from their good health in adulthood are not fully reflected in the preferences of today's parents, who are assumed to make the relevant decisions in the present. If, as is arguable, the government has – or should have – a longer horizon than individual households, then the case for intervention to promote schooling at the expense of child labor and to lower premature adult mortality by combating the disease is, in principle, established.

Second, communicable diseases have a strong public good aspect, which calls for public intervention on standard grounds. This argument is reinforced when the disease reduces

the returns to investment in human capital. Third, when there is asymmetric information regarding the consequences of a disease and how to prevent it, a case can be established for public campaigns that provide information on how to protect against, and to deal with, the disease – and for the provision of the right incentives to undertake such measures. Fourth, as discussed from section 3.4 of the paper onwards, more than one time path – self-sustaining growth and enduring poverty are two possibilities – can exist under the same set of policies. These arguments provide strong justifications for governments to promote education and to combat diseases, especially the communicable kind like AIDS, and to do so in a way that makes future policy credible.

The instruments available to the government to attain the broad objectives of averting a collapse and ensuring the conditions for self-sustaining growth in a disease-ridden environment are of three kinds:

- (i) subsidies designed to encourage education;
- (ii) spending on measures to combat the spread of the disease and to treat those infected by it; and
- (iii) raising the taxes needed to finance these expenditures.

The associated policies are now discussed in greater detail, whereby we concentrate on the case of nuclear families.

3.1.1 Education Policy

The direct promotion of education takes the form of subsidies to households. These are paid either as general transfers or, more efficiently, conditional on the children attending school. They are financed by taxes on income, where it should be noted that a household's ability to

pay depends on its state. For simplicity, therefore, we introduce

Assumption 9. The government can identify both a household and its state. Only the income of (healthy) adults is taxable.⁶

Let $\tau_t(\Lambda_t^i(s_t))$ denote the tax levied in period t on household i if it is in state s_t . Some fraction of the population will be subsidized out of the ensuing revenues. Starting with general transfers, we denote by $g_t^i(\Lambda_t^i(s_t))$ the subsidy household i will receive in period t in state s_t , where subsidies should be interpreted in a broad sense; for instance, they may take the form of supporting local infrastructure. We denote by $w_t^i(s_t)$ the net income of household i in state s_t in period t , measured in units of output:

$$w_t^i(s_t) = \alpha \Lambda_t^i(s_t) + \alpha n_t(1 - e_t^i)\gamma + g_t^i(\Lambda_t^i(s_t)) - \tau_t(\Lambda_t^i(s_t)) \equiv w_t^{ia}(s_t) + \alpha n_t(1 - e_t^i)\gamma \quad (19)$$

where w_t^{ia} denotes the adults' total net disposable income. The household's net tax burden in state s_t is defined as

$$v_t^i(\Lambda_t^i(s_t)) \equiv \tau_t(\Lambda_t^i(s_t)) - g_t^i(\Lambda_t^i(s_t)). \quad (20)$$

Although the family chooses e_t^i on the basis of its potential full income after tax (equivalently, on w_t^{ia}), it is important to note that an increase in λ_t not only enlarges the feasible set in the space of (c_t, λ_{t+1}) , but also make its upper boundary steeper. A decrease in the net tax burden, however, will simply shift the said boundary to the right. Notice also that subsidization can be made dependent on income *and* the household's identity, i.e. on the index i . The optimal educational choice is therefore written as $e_t^{i0}(\Lambda_t^i(s_t), \tau_t(\Lambda_t^i(s_t)) - g_t^i(\Lambda_t^i(s_t)), s_t, \kappa_{t+1})$.

If, instead, subsidies are payable only on condition of school-attendance, then household i 's budget line becomes

$$[(3 - s_t) + n_t\beta]c_t^i(s_t) + n_t \cdot [\alpha\gamma - \sigma_t^i(\Lambda_t^i(s_t))]e_t(s_t) = \alpha[(3 - s_t)\lambda_t^i + n_t\gamma], \quad s_t = 1, 2 \quad (21)$$

⁶Taxing only adults may be justified by the easiness of tax evasion for child income. It is unlikely that allowing household income to be taxable would change the main results of the paper.

where $\sigma_t^i(\Lambda_t^i(s_t))$ is the subsidy payable to the family for each unit of time each child spends at school, and it will be recalled from (2) that the states $s_t = 2$ and $s_t = 3$ are identical in this respect.

Where ability to pay is concerned, we assume that there is a subsistence level c^{sub} for each adult and βc^{sub} for each child which must be ensured under all circumstances. Allowing for the possibility that not all households with the same characteristics will receive subsidies, household i 's tax burden is therefore constrained by:

$$\alpha[(3 - s_t)\lambda_t^i + n_t\gamma] - \tau_t(\Lambda_t^i(s_t)) \geq [(3 - s_t) + n_t\beta]c^{sub} \quad s_t = 1, 2 \quad (22)$$

where it is assumed that when $\lambda_t^i = 1$ and the household receives no subsidies, all children work full-time.

In order to make possible an escape from what we have termed a general state of backwardness without outside help, we allow for some limited ability to pay whenever $\lambda_t^i = 1$. In particular, the tax schedule for single-parent households must fulfill the condition

$$0 \leq \tau_t(1(s_t)) \leq \alpha(1 + n_t\gamma) - (1 + n_t\beta)c^{sub} \equiv \tau^{ba}, \quad (23)$$

where it is plausible that τ^{ba} is small. Note that under the innocuous assumption that $\alpha > c^{sub}$, a two parent-family is also able to pay at least τ^{ba} .

3.1.2 Health Policy

Health policy takes the form of spending on measures to combat the disease. Here, we distinguish not only between prevention and treatment, but also between expenditures that produce private and public goods. For some diseases, treatment may result in a complete cure. There is no such prospect for the victims of AIDS; but the treatment of opportunistic infections in the later stages and the use of anti-retroviral therapies (ART) can prolong life and maintain productivity. In the present OLG setting, therefore, treatment may be thought

of as reducing premature adult mortality in the probabilistic sense. The distinction between private and public goods is also an idealization, and a hard one to draw where communicable diseases are concerned. Regular exercise and good diet, for example, will lower the chances of heart disease only among those individuals who take this prescription seriously. Washing one's hands often, or staying at home when suffering from influenza, however, reduces the chances of passing on infection to others. Large-scale public programs aimed at combating the spread of communicable diseases are therefore society-wide programs from which everybody can benefit and large externalities are present. Examples are public awareness campaigns and, in the case of AIDS, the provision of condoms at no charge. The public good nature of such health policies stems from the positive externalities that arise when other individuals benefit from the efforts of an individual to lower his or her risk of getting infected, or, if already infected, the risk of also infecting others.

Formulating the effects of health policy is fairly straightforward in the 'pure' cases. Let the probabilities that the adults (father and mother) in family i die prematurely be denoted by q^{fi} and q^{mi} , respectively: the nature of AIDS being what it is, these are not independent events, and we treat them as such only in section 5. If spending on prevention, broadly construed, produces a purely private good for the individual in question, then

$$q^{li} = q^l(h_t^{li}, \Lambda_t^i), \quad l = f, m \quad (24)$$

where h_t^{li} is the amount spent on the individual in question and the possibility that the couple's combined human capital upon forming the family may influence such mortality is allowed.

At the other extreme, let spending on prevention produce a pure public good, where it is possible that the size of the population may affect the efficacy of spending. For simplicity, we assume that the causes of premature adult mortality are no respecters of human capital, so that we may write

$$q^l = q^l(\eta_t, N_t), \quad l = f, m \quad (25)$$

where the number of families in period t is denoted by N_t and η_t denotes the level of spend-

ing per family. The measure of all families at the start of period 0 will be normalized to unity. The function $q^l(\cdot)$ is assumed to have the following properties:

Assumption 10 $q^l(0, \cdot) = \bar{q}(< 1)$; $\lim_{\eta_t \rightarrow \infty} q^l(\eta_t, N_t) = \underline{q}$; $\partial q^l / \partial \eta_t < 0$; $\partial^2 q^l / \partial (\eta_t)^2 > 0$.

The same assumptions hold for $q^{li} = q^l(h_t^{li}, \Lambda_t^i)$. As indicated above, the treatment of diseases prolongs productive life, even if no cure is available. When the disease is also communicable, substantial external benefits will often result. In the framework adopted here, the ensuing private and external benefits will be represented as a reduction in premature adult mortality.

3.1.3 The Government's Budget Constraint

In formulating the budget constraint, some care is needed in distinguishing between the level of an adult's human capital and the state of the family to which he or she belongs. For simplicity – and this will indeed hold in all the settings analyzed in the remainder of the paper –, let there be a discrete distribution of the levels of individual adults' human capital in any period t , the vector of which is denoted by $[\lambda_t^1, \dots, \lambda_t^{m(t)}]$, where the elements are arranged in ascending order and the number of such classes in period t is denoted by $m(t)$. The vector of corresponding measures, normalized by the population of individuals who have just reached adulthood, is denoted by $[\mu_t^1, \dots, \mu_t^{m(t)}]$, with representative element μ_t^k and $\sum_{k=1}^{m(t)} \mu_t^k = 1$.

Allowance must also be made for the fact that the population may be growing. Without loss of generality, let the measure of all families at the start of period 0 be unity. By assumption 5, therefore, the measure of families at the start of period $t(> 0)$ is

$$N_t = n_0 n_1 \dots n_{t-1} / 2^t = \frac{1}{2^t} \prod_{k=1}^t n_{k-1}.$$

Consider the group of adults with human capital λ_t^k . Recalling assumption 9, their aggregate

tax payments are

$$B_t^k = [\pi(1)\tau_t(\Lambda_t^k(1)) + (\pi(2) + \pi(3))\tau_t(\Lambda_t^k(2))]\mu_t^k \cdot N_t.$$

The specific expenditures on this group require further differentiation; for some members may receive subsidies and others none at all. In order to allow for this differentiation, and with a slight abuse of notation, let each family in group k be indexed by $i_k \in [0, 1]$. With a slight abuse of notation, the aggregate, specific expenditure on group k is, therefore,

$$\left[\int_0^1 (g_t^{i_k}(\Lambda_t(s_t)) + h_t^{fi_k} + h_t^{mi_k}) di_k \right] \mu_t^k \cdot N_t.$$

Hence, the government's budget constraint may be written

$$\begin{aligned} B_t &\equiv \left[\sum_{k=1}^{m(t)} \sum_{s_t=1}^3 \pi(s_t) \tau_t(\Lambda_t^k(s_t)) \mu_t^k \right] \cdot N_t + \bar{B}_t \\ &\geq \left[\sum_{k=1}^{m(t)} \left(\int_0^1 (g_t^{i_k}(\Lambda_t(s_t)) + h_t^{fi_k} + h_t^{mi_k}) di_k \right) \mu_t^k + \eta_t \right] \cdot N_t, \end{aligned} \quad (26)$$

where \bar{B}_t denotes the fiscal resources, such as foreign aid, arising from outside the system.

Two remarks on the relationship between (24) and (25) are called for. First, adult mortality in period t depends on the level of spending on measures to combat the disease in period t . Since revenues other than \bar{B}_t depend, in turn, on the level of mortality, there is an apparently odd form of simultaneity here: many of those individuals who survive the disease do so because their survival provided part of the tax base, expenditures out of which kept them alive. Unless \bar{B}_t is so large as to permit the government to finance any desired package of measures in the health domain, the indivisibility of a period and the impossibility of ‘storage’ therefore leave one with an awkward difficulty. This can be overcome, however, by artificially splitting each period into two. Suitably stiff taxes are levied in the first sub-period in order to finance measures in the health domain and to provide initial support for needy children, followed by comparatively more modest taxation in the second, as the children grow up. The second point concerns the relation between the efficacy of spending in the health domain to produce

public goods and the size of the adult population. Given the nature of AIDS and of human behavior, it is plausible that a doubling of the population would require roughly a doubling of spending in order to attain the same level of premature adult mortality, and this specialization of (24) is incorporated into (25) above.

3.2 The Efficacy of a Single Policy Instrument

In this section, we discuss the case where the government is unable or unwilling to intervene in both the health and the education domains at the same time. The analysis of this case will not only yield insights into the costs imposed by such a limitation, but it will also be helpful in the design of a comprehensive policy program, in which the interplay of both kinds of interventions plays an important role.

3.2.1 Education Policy

We ask what can be accomplished through subsidies alone, taking the disease environment, represented by the vector $[\pi(1), \pi(2), \pi(3), \pi(4)]$, as given and constant following the outbreak of the disease in period 0. No measures are undertaken to combat the disease, presumably because they are deemed to be ineffective.

Our first result is that even when the government is limited to promoting education, there are conditions under which a long-run collapse can be avoided. In order to show this, we assume that the society is in a state of backwardness when it is assailed by the disease. The argument then holds for any starting value $\lambda_0 \geq 1$.

We begin with the following set of assumptions, under which an escape from backwardness may be possible, even in the face of premature adult mortality.

Assumption 11.

$$\alpha[z(2)f(1)\Lambda^a(2, \kappa) + 1] - \alpha\Lambda^a(2, \kappa) \geq \tau^{ba}$$

The assumption states that if a child reared in a single-parent family whose adult possesses human capital $\Lambda^a(2, \kappa)$ is fully educated, then that child, on becoming a single parent, can pay at least τ^{ba} while also choosing to educate his or her own children fully. Recall that by definition, such a child will indeed receive a full education if the family pays no net taxes.

Assumption 11 is equivalent to

$$\alpha[(z(2)f(1) - 1)\Lambda^a(2, \kappa) + 1] \geq \tau^{ba} \quad (27)$$

Since $z(2)f(1) \geq 1$ is a necessary and sufficient condition for unbounded growth to be possible in the absence of premature adult mortality, and since $\Lambda^a(2, \kappa) > \lambda_0 = 1$, inspection of (22) reveals condition (27) to be a very modest requirement.

Assumption 12. There exists a positive, bounded subsidy \bar{g} that satisfies

$$z(2)f\left[e^0(1, \tau^{ba} - \bar{g}, 2, \kappa)\right] \cdot 1 + 1 = \Lambda^a(2, \kappa) + \tau^{ba}/\alpha \quad (28)$$

Since the net income of adults in a household $\alpha + \bar{g} - \tau^{ba}$ matters for the choice of education, e^0 is now written as a function of net income. This assumption states that the combination of the educational technology and the transmission factor $z(2)$ is strong enough to yield a bounded transfer \bar{g} (gross of the tax τ^{ba}) that will induce a single-parent family whose adult human capital is unity to choose an $e^0 \in (0, 1)$ such that its children will attain the level of human capital $\Lambda^a(2, \kappa) + \tau^{ba}/\alpha$. Note that \bar{g} is a gross subsidy since the single-parent family pays taxes τ^{ba} .

Finally, we appeal to the existence of satisfactory institutional arrangements for the care and education of full orphans, whereby the cost per child may be larger than that corresponding to the family transfer \bar{g} . The following assumption allows, in principle, a solution to this social problem if fiscal resources are large enough over some sequence of periods.

Assumption 13. Full orphans can be supported in such a way that they can attain a level of human capital of at least $\Lambda^a(2, \kappa)$ as adults.

In order to complete the preliminaries, we introduce some additional notation. Let $\delta_t \in [0, 1]$

denote the share of all N_t families that receive the gross subsidy \bar{g} in period t . Moreover, let $\delta_t(s_t) \in [0, 1]$ denote the corresponding share of all households of type s_t that receive some form of support in period t .

The following result establishes that all children can be fully educated from some point in time onwards.

Proposition 3

Suppose a society in a general state of backwardness is assailed by an epidemic at the start of period 0, which induces the state-vector $[\pi(1), \pi(2), \pi(3), \pi(4)]$ from then onwards. Suppose also that assumptions 11 – 13 hold. Then, if $(1 - \pi(4))z(2)f(1) > 1$, there exists a sequence of taxes and transfers such that all individuals will enjoy full education within a finite number of periods.

The proof, which is constructive, is given in the appendix. It should be noted that if, on the contrary, $(1 - \pi(4))z(2)f(1) < 1$, the tax revenues from all groups ‘promoted’ to the condition $e = 1$ will eventually grow more slowly than the population, given the requirement that both one- and two-parent families are to choose $e = 1$ after promotion. Since the fraction $(1 - \pi(4))$ of all children become full orphans in each period, it is not clear that all can be fully educated when the condition $(1 - \pi(4))z(2)f(1) > 1$ does not hold.

Turning to the effects of premature adult mortality on the behavior of output over the long run, it is obvious that aggregate output in every period is strictly greater in the complete absence of such mortality than in its presence. What is of keener interest, however, is to establish whether an increase in such mortality reduces the long-run rate of growth of output below the value $[2z(1)f(1) - 1]$, which is the rate that would obtain if there were no premature adult mortality at all.

In order to answer this question, we begin by noting from proposition 3 that there is a policy under which all children will enjoy full-time schooling by the start of period t , say. At this juncture, there will be a discrete distribution of the levels of individual adults’ human

capital, the vector of which is denoted by $[\lambda_t^1, \dots, \lambda_t^{m(t)}]$, where the elements are arranged in ascending order and, in view of assumption 13, $\lambda_t^1 = \Lambda^a(2, \kappa)$. The vector of corresponding measures, normalized by the population of individuals who have just reached adulthood, is denoted by $[\mu_t^1, \dots, \mu_t^{m(t)}]$, where $\mu_t^1 = \pi(4)$. Recalling (3), the average human capital of their children when the latter reach adulthood at the start of period $t + 1$ is

$$K_{t+1} = \pi(1) \left[2z(1)f(1) \sum_{k=1}^{m(t)} \mu_t^k \lambda_t^k + 1 \right] + [\pi(2) + \pi(3)] \left[z(2)f(1) \sum_{k=1}^{m(t)} \mu_t^k \lambda_t^k + 1 \right] + \pi(4)\Lambda^a(2),$$

where $K_t = \sum_{k=1}^{m(t)} \mu_t^k \lambda_t^k$ is the corresponding average at the start of period t . Hence, the growth rate of this aggregate in period t is

$$\frac{K_{t+1}}{K_t} - 1 = [\pi(1)2z(1) + (\pi(2) + \pi(3))z(2)]f(1) - 1 + \frac{1 - \pi(4)}{K_t} + \pi(4)\frac{\Lambda^a(2)}{K_t}. \quad (29)$$

If $[\pi(1)2z(1) + (\pi(2) + \pi(3))z(2)]f(1) > 1$, this aggregate will grow without limit. By virtue of assumption 2, this is a weaker condition than $[1 - \pi(4)]z(2)f(1) > 1$ whenever $z(1) > z(2)/2$. Since we are concerned with a comparison of long-term growth rates, we therefore assume that $[\pi(1)2z(1) + (\pi(2) + \pi(3))z(2)]f(1) > 1$. It is clear from (28) that K_t grows at the asymptotic rate of

$$[\pi(1)2z(1) + (\pi(2) + \pi(3))z(2)]f(1) - 1 < 2z(1)f(1) - 1$$

whenever there is any premature adult mortality.

Now, this result is an immediate consequence of assumption 13, which implies that a fixed proportion $\pi(4)$ of all adults will attain the (fixed) level of human capital $\Lambda^a(2, \kappa)$ at the start of each period after period t . We therefore modify assumption 13 conditionally as follows:

Assumption 13(a). Under conditions of long-run growth in productivity, full orphans in period t can be supported in such a way that they attain ξK_{t+1} as adults in period $t + 1$, where $\xi \leq 1$.

It should be remarked that it is very plausible that full orphans do rather less well, on average, than their cohort: equivalently, that $\xi < 1$. Be that as it may, it follows from (28) and assumption 13(a) that the asymptotic rate of growth is⁷

$$\frac{[\pi(1)2z(1) + (\pi(2) + \pi(3))z(2)]f(1)}{[1 - \xi\pi(4)]} - 1$$

which is less than $2z(1)f(1) - 1$. Recalling assumption 2, we have obtained the following result:

Proposition 4

If $[\pi(1)2z(1) + (\pi(2) + \pi(3))z(2)]f(1) > 1 - \xi\pi(4)$ and $(1 - \pi(4))z(2)f(1) > 1$ under the fixed state-vector $[\pi(1), \pi(2), \pi(3), \pi(4)]$, then unbounded long-run growth is feasible. Any premature adult mortality will result in a lower rate of growth of human capital, and hence of output, over the long run if either of the conditions $\xi = 1$ and $z(1) = z(2)/2$ is violated. There will be no such reduction if and only if both $\xi = 1$ and $z(1) = z(2)/2$ hold.

It is very plausible that premature adult mortality has an adverse effect on the growth rate of aggregate output, even when the condition of long-run growth remains attainable. This condition may not be reached at all, however, if the condition $(1 - \pi(4))z(2)f(1) > 1$ is strongly violated; for then the condition of universal full education, on which the above argument rests, may not be attainable. High premature adult mortality, as expressed in a large value of $\pi(4)$, may therefore destroy all prospects of long-run economic growth, even when $[\pi(1)2z(1) + (\pi(2) + \pi(3))z(2)]f(1) > 1 - \xi\pi(4)$, $\xi = 1$ and $z(1) = z(2)/2$ all hold.

Although propositions 3 and 4 suggest there are conditions under which concentrating resources on the promotion of education can yield sustainable growth of human capital at an

⁷The details of the derivation are available upon request.

asymptotically steady rate, it is generally the case that allocating at least some resources to health is advantageous. Finding the right balance will be taken up shortly, but we first look briefly at the other polar case, in which all resources are devoted to improving health, in the sense of reducing premature adult mortality.

3.2.2 Health Policy

The first fact is obvious.

Lemma 5

Suppose $\underline{q} > 0$, so that $\kappa < 1$. If there is a pure nuclear family structure, then for any level of λ_0 , health policy alone can delay, but cannot avert a collapse, i.e. $\lim_{t \rightarrow \infty} P_t = 1$.

The lemma follows immediately from the second part of proposition 1, which holds for any level of $\kappa < 1$. If health policy cannot eliminate premature mortality completely, the society slides back into backwardness over time. It would be a grave mistake, however, to put too much weight on the significance of lemma 5. Under other circumstances, health policy alone can avert a collapse: the obvious example is full-scale pooling, which we will take up later. But there are also more subtle and indirect forces which slow the descent into general backwardness. For instance, as long as the share of orphans remains small, a majority of the society may opt for a program of compulsory schooling of orphans, a measure which would stave off a long-term collapse.

3.3 Double versus Single Targeting

In this section we investigate how fiscal resources should be allocated between education and health, and how these resources should be concentrated on subgroups of the population when both types of policies are deployed. In particular, we examine whether health and education support should be given simultaneously to subgroups of the society, at least initially.

We distinguish between what we call double and single targeting. Under double targeting, some needy families receive both the benefits of spending on health and transfers to promote education. Under single targeting, such families receive transfers at the start, and the benefits of health spending later. We discuss the cases of private and public goods in that order.

3.3.1 Health Spending is a Private Good

Since the most general case is extremely complex, we confine ourselves to a representative example. Recalling assumption 12, observe that if \bar{g} is paid to a two-parent family in period 0 ($\Lambda_0(1) = 2$), then its children will attain

$$\lambda_1 = z(1)f(e^0(2, \tau^{ba} - \bar{g}, 1)) \cdot 2 + 1 \geq z(2)f(e^0(1, \tau^{ba} - \bar{g}, 2)) \cdot 1 + 1 = \Lambda^a(2, \kappa) + \frac{\tau^{ba}}{\alpha}.$$

Denote by $\bar{\tau}$ the largest tax that a two-parent household formed by such a child on reaching adulthood could pay while still choosing $e_1 = 1$ for its children. It is clear that $\bar{\tau} \geq \tau^{ba}$, where the equality holds if and only if $z(1) = z(2)/2$ and $e^0(1, \tau^{ba} - \bar{g}, 2) = 1$.

When spending on health produces a purely private good, in the form of a lower probability of premature death for a particular adult, we specialize $q^l(h^{li}, \cdot)$ as follows:

Assumption 14.

$$1 - q^l(h^{li}, \cdot) = \begin{cases} 1 & \text{if } h^{li} \geq \bar{h}, \\ p & \text{otherwise} \end{cases}$$

The constant ($p < 1$) is the probability that an adult male or female will survive to old age if spending on him or her should fall below \bar{h} . Allowing spending in the amount \bar{h} to eliminate the risk of premature mortality altogether is a convenient simplification. Note that such mortality is independent across individuals in the present setting; for $h^{li} < \bar{h}$, it is also identically distributed. Hence, for all $p \in [0, 1]$,

$$[\pi(1), \pi(2), \pi(3), \pi(4)] = [p^2, p(1-p), p(1-p), (1-p)^2].$$

We concentrate on the following comparison of double targeting (DT) and single targeting (ST) when the society is initially in a state of general backwardness. In period $t = 0$ one of the two schemes is put into effect. In $t = 1$, the scheme chosen is completed for the families targeted in $t = 0$, while DT is applied to the next targeted generation. In all subsequent periods, we assume that an optimal scheme is used, but without specifying precisely how that scheme operates. This allows us to concentrate on the comparison of DT and ST within the first two periods.

The following proposition states a sufficient condition for ranking them:

Proposition 5

Suppose that a society in a state of general backwardness ($\lambda_0 = 1$) is assailed by a non-communicable disease in $t = 0$. Suppose that assumptions 12 and 14 hold and that $\bar{\tau} > 2\bar{h}$. Then, in the setting described above, DT is superior to ST if

$$\frac{\bar{g}}{\bar{h}} > \frac{4p - 2p^2}{(1 - p)^2} \quad (30)$$

The proof is given in the appendix. Observe that condition (30) is necessary as well as sufficient if and only if both $z(1) = z(2)/2$ and $e^0(1, \tau^{ba} - \bar{g}, 2) = 1$; for only then does $\bar{\tau} = \tau^{ba}$ hold.

Proposition 5 has an intuitive interpretation. The higher the costs of promoting education relative to those of reducing premature adult mortality, the more attractive is DT, since it is relatively cheap to prevent the early death of adults educated in period zero. If, in contrast, the costs of preventing premature mortality are relatively high, it is more efficient to promote education alone. The higher is such mortality, the more attractive DT becomes relative to ST, since the waste of resources devoted to education becomes larger.

3.3.2 Health Spending is a Public Good

The preceding analysis can be adapted in a straightforward manner to the case where spending on health produces a public good. Analogous to assumption 14, we have

Assumption 15. Spending at least the aggregate amount $\bar{\eta}N_t$ on public health in period t eliminates the disease in that period; spending less has no effect on premature mortality, whose profile is exogenously given. That is,

$$[\pi_t(1), \pi_t(2), \pi_t(3), \pi_t(4)] = \begin{cases} [1, 0, 0, 0] & \text{if } \eta_t \geq \bar{\eta} \\ [\pi(1), \pi(2), \pi(3), \pi(4)] & \text{otherwise} \end{cases} \quad (31)$$

We then obtain:

Proposition 6

Suppose a society in a general state of backwardness is assailed by an epidemic at the start of period 0. Suppose also that fiscal resources in period 0 exceed $\bar{\eta}$, and that assumptions 12 and 15 hold. Outside aid, if any, is small and available only in that period. Then DT is superior to ST if

$$B_0 > \max \left\{ \frac{[\bar{\eta} - (1 - \pi(4))\tau^{ba}]\bar{g}}{\pi(1)(\bar{\tau} - \tau^{ba})}, \frac{\bar{\eta}[\bar{g} + (\bar{\tau} - \tau^{ba})] - \pi(4)\tau^{ba}\bar{g}}{\pi(4)\bar{g} + (1 - \pi(1))(\bar{\tau} - \tau^{ba})} \right\}. \quad (32)$$

The proof is given in the appendix.

To gain further insights into how the various factors influence the outcome, consider the special case where $\bar{\tau} = \tau^{ba}$. If $\bar{\eta} \leq (1 - \pi(4))\tau^{ba}$, then condition (31) specializes to

$$B_0 = \tau^{ba} + \bar{B}_0 > (\bar{\eta}/\pi(4)) - \tau^{ba},$$

where \bar{B}_0 is the amount of outside aid in period 0. The condition is equivalent to:

$$2\tau^{ba} + \bar{B}_0 > (\bar{\eta}/\pi(4)),$$

which is independent of \bar{g} . In the absence of outside aid ($\bar{B}_0 = 0$), the said condition reduces further to $2\pi(4)\tau^{ba} > \bar{\eta}$. Hence, DT is superior to ST for all $\pi(4) \in \left(\frac{\bar{\eta}}{2\tau^{ba}}, 1 - \frac{\bar{\eta}}{\tau^{ba}}\right]$.

The intuition is clear: the more orphans the disease creates, the more attractive is DT; for under the above assumptions, DT eliminates the disease, and so preserves in period 1 all the investment in education in period 0. Foreign aid in period 0, when public funds are likely to be especially scarce, also works in favor of DT. This general conclusion provides a foundation for using health policy from the start in the formulation of good policy programs.

3.4 Multiple Equilibria, Self-Fulfilling Prophecies and the Credibility of Policies

Our model exhibits an important difference between education and health policies where the role of expectations is concerned. While measures to promote education in the future do not affect schooling choices in the present, measures that promote health in the future affect the well-being of today's children on reaching adulthood and thus influence parents' schooling choices today. As a consequence, parents' decisions depend on their expectations about future health policies, which, in turn, may depend on the education choices of all families today, since the latter determine tax revenues in the future. It is not surprising, therefore, that the model allows multiple equilibria and the possibility of self-fulfilling prophecies. In particular, we may have time paths that lead either to a progressive collapse, or to literacy, health and growth under the same policy schemes, whereby the expectations of agents turn out to be correct along all possible paths. The reason is that the government's ability to combat the disease in the present depends on current tax revenues, which depend, in turn, on parents' education choices one period earlier, which depended, in turn, on the parents' expectations about the government's ability to undertake the necessary measures to suppress the disease in the present. Hence, both vicious and virtuous circles are possible.

Since the existence of such multiple equilibria is an important concern in the design of policy,

and the credibility of future policies then becomes crucial, we identify in this section the most important source of multiple equilibria and discuss potential ways of responding to the resulting problems.

In the following continuation of the events following the outbreak of the epidemic in period 0, there is a lag in the government's response. The epidemic claims its victims early on in adulthood before any effective measures can be devised to combat it, so that the (fixed) state-vector $[\pi(1), \pi(2), \pi(3), \pi(4)]$ prevails until period 1. From that point onwards, assumption 15 holds, and the government formulates a spending program and the tax structure to finance it; there are no outside resources. Spending $\bar{\eta}N_t$ on health, if at all feasible, takes priority over transfers in all periods. Such a program will be called henceforth a *PHE*, for “priority for health expenditures”.

In view of the heterogeneity that necessarily arises in period 0, when all adults start with λ_0 , and is perpetuated in period 1, it will be useful to introduce some additional notation. Define

$$\lambda_1(s_0, \kappa_1) \equiv z(s_0)f(e_0^0(\Lambda_0(s_0), s_0, \kappa_1))\Lambda_0(s_0) + 1, \quad s_0 = 1, 2, 3$$

$$\lambda_2(s_1, \kappa_2; s_0, \kappa_1) \equiv z(s_1)f(e_1^0(\Lambda_1(s_1; s_0, \kappa_1), s_1, \kappa_2))\Lambda_1(s_1; s_0, \kappa_1) + 1, \quad s_1 = 1, 2, 3$$

Observe that if $\kappa_1 = 1$, then only $s_1 = 1$ occurs. The next step is to define taxable capacity in relation to c^{sub} , as in section 3.1. Recalling that the state-vector $[\pi(1), \pi(2), \pi(3), \pi(4)]$ prevails in period 0, define taxable capacity in relation to (23), normalized by the number of families, such that full education is still feasible as

$$\begin{aligned} \Theta_1(\kappa_1) \equiv & \pi(1)\{\alpha[\pi(1)2\lambda_1(1, \kappa_1) + (\pi(2) + \pi(3))\lambda_1(1, \kappa_1)] \\ & - [\pi(1)(2 + n_1\beta) + (\pi(2) + \pi(3))(1 + n_1\beta)]c^{sub}\} + \\ & (\pi(2) + \pi(3))\{\alpha[\pi(1)2\lambda_1(2, \kappa_1) + (\pi(2) + \pi(3))\lambda_1(2, \kappa_1)] \\ & - [\pi(1)(2 + n_1\beta) + (\pi(2) + \pi(3))(1 + n_1\beta)]c^{sub}\} \end{aligned}$$

Since $\sum_{s=1}^4 \pi(s) = 1$, some rearrangement yields

$$\Theta_1(\kappa_1) \equiv \alpha \cdot 2\kappa_1[\pi(1)\lambda_1(1, \kappa_1) + (\pi(2) + \pi(3))\lambda_1(2, \kappa_1)] - (1 - \pi(4))[2\kappa_1 + (1 - \pi(4))n_1\beta]c^{sub}.$$

In keeping with the question posed, we assume that $2z(1)f(1) > 1$, so that unbounded growth per capita will occur if $\eta_t \geq \bar{\eta}$ for all $t > 0$. The following assumption states that if parents expect the aggregate level of spending on health in period 1 to fall short of $\bar{\eta}N_1$, then the level of premature mortality in that period, as represented by the statistic κ_1 , is so great that they will choose to educate their children in such a limited way that no child will attain the corresponding critical value $\lambda^*(s_0, \kappa_1)$ in period 1. Expressed formally, we have

Assumption 16.

$$\lambda_1(s_0, \kappa_1) < \lambda^*(1, 1), \quad s_0 = 1, 2, 3.$$

Here, it should be recalled from part (ii) of lemma 2 that $\lambda^*(1, \kappa) \leq \lambda^*(2, \kappa) = \lambda^*(3, \kappa)$. The next assumption states that if, on the contrary, parents in period 0 expect the disease to be suppressed in period 1, and hence that none of their children will die prematurely as adults, then all will be so educated as to exceed the corresponding critical value when they reach adulthood in period 1.

Assumption 17.

$$z(s_0)f(e_0^0(\Lambda_0(s_0), s_0, 1))\Lambda_0(s_0) + 1 \geq \lambda^*(2, 1), \quad s_0 = 1, 2, 3.$$

The following proposition establishes a sufficient condition for the existence of two, self-fulfilling expectations equilibria.

Proposition 7

In the setting described above, let the government commit itself to PHE. Suppose that assumptions 16 and 17 hold, and that

$$\Theta_1(\kappa_1 < 1) < \bar{\eta} < \sum_{s_0=1}^3 \alpha \pi(s_0) \cdot [\Lambda_1(s_0, 1) - \Lambda^*(s_0, 1)] \quad (33)$$

Then, under rational expectations, there exist both

- (i) *a time path along which the whole society slides into the poverty trap, and*

(ii) a time path along which the disease is fully suppressed and the whole society enjoys unbounded growth at the asymptotically steady rate $2z(1)f(1) - 1$.

Proof: see appendix.

If $\bar{\eta}$ is not too large, condition (33) can always be ensured by choosing κ_1 sufficiently close to zero.

Several remarks are in order. The multiplicity of paths a society can experience arises because the government's ability to combat the disease depends on current tax revenues, which depend, in turn, on the expectations of adults one period earlier about whether the government will be able to combat the disease effectively. Although the government is able to commit to a policy scheme, its inability to commit to a specific policy, such as spending at the level $\bar{\eta}N_1$, creates the possibility of self-fulfilling prophecies. As an immediate consequence we obtain:

Corollary 1

Suppose that the government can commit to spend $\bar{\eta}N_1$ on combating the disease in $t = 1$. Then, in the setting described in proposition 7, there exists a unique time path along which the society enjoys self-sustaining growth.

The corollary illustrates the point that the government must be able to tap sources of revenues other than current taxation, such as outside aid or world capital markets, to combat the disease if that measure would avoid a vicious circle leading to a wholesale collapse. To be effective such announcements must be credible.

4 Fairly Good Policy Programs

After these extensive preparations, we address not only the most important, but also the most difficult question confronting decision-makers: namely, what does the optimal sequence of

expenditure and tax policies look like? Rather than attempting to characterize such programs, as is done in the related, but far simpler, setting of Bell and Gersbach (2002), we approach the problem in the spirit of a strand in the literature on economic planning. This sought to formulate and describe what were called ‘fairly good plans’, as a way of dealing with the deeply intractable problem of finding a full optimum. The results obtained here will serve as the basis for the analysis of the South African case in section 5.

As in previous sections, we begin with a homogeneous society at the start of period 0, which is then struck by an epidemic. In keeping with the nature of the AIDS epidemic, public spending on measures to combat it are treated as a public good, as in section 3.3.2. The pre-existing social structure also has a profound influence on the right policy response. We begin with nuclear families, and then turn to pooling.

4.1 Nuclear Families

A good response to the outbreak must cover two phases. The first involves an attempt to re-establish the conditions for sustainable growth after the dislocations caused by the initial shock. The second involves maintaining those conditions in the face of a continuing epidemic. In other words, the first phase is concerned with the so-called ‘traverse’ to a new steady state and the second with the new steady state to which the traverse leads.

In the present context, one principal concern is with bringing about full schooling for all. With this goal in mind, we define the taxable capacity of a household of type s_t whose adults possess human capital $\Lambda_t(s_t)$ as the largest possible tax that can be imposed on it without violating $e_t^0(\Lambda_t(s_t), \tau_t(s_t), s_t, \kappa_{t+1}) = 1 \ \forall s_t \neq 4$. This tax is denoted by

$$\bar{\tau}_t(\Lambda_t(s_t), s_t, \kappa_{t+1}) = \max\{0, \alpha(\Lambda_t(s_t) - \Lambda^a(s_t, \kappa_{t+1}))\} \quad s_t = 1, 2, 3. \quad (34)$$

Observe that $\bar{\tau}(\cdot)$ is increasing in $\Lambda_t(s_t) \ \forall \Lambda_t(s_t) \geq \Lambda^a(s_t, \kappa_{t+1}) \ \forall s_t \neq 4$, and that for a given level of the household’s human capital and κ_{t+1} , $\bar{\tau}_t(s_t = 1) > \bar{\tau}_t(s_t = 2) = \bar{\tau}_t(s_t = 3)$. Part

(iii) of lemma 1 implies that higher premature mortality in period $t + 1$, as expressed by a decrease in κ_{t+1} , will decrease $\bar{\tau}_t \forall \Lambda_t(s_t) > \Lambda^a(s_t, \kappa_{t+1}) \forall s_t \neq 4$.

Where the relationship between premature adult mortality and public spending on measures to combat the disease is concerned, we relax the special form in assumption 15 as follows:

Assumption 18. The probability $\pi_t(s_t)$ is a continuous and differentiable function of the spending level η_t with:

$$\pi_t(s_t) = \pi_t(\eta_t, s_t), \text{ where } \pi'_t(\eta_t, 1) > 0, \pi'_t(\eta_t, s_t) < 0 \forall s_t \neq 1.$$

The direct dependence of $\pi_t(s_t)$ on t allows for the possibility that the disease environment may change over time, for example, following the outbreak of a new disease. Since $\sum_{s_t=1}^4 \pi_t(\eta_t, s_t) = 1$, assumption 18 also implies that

$$\sum_{s_t=1}^3 \pi'_t(\eta_t, s_t) = -\pi'_t(\eta_t, 4) > 0. \quad (35)$$

An additional implication of assumption 18 is that κ_t is an increasing, continuous and differentiable function of the spending level η_t , as can be seen at once from (8).

4.1.1 The Second Phase

We begin with the second phase of this policy program, since it is the easier of the two to analyze. Consider, therefore, the situation in which, by hypothesis, the initial shock has been overcome, in the following sense: at the start of period t , all adults are endowed with at least $\lambda^a(2, \kappa_{t+1})$ units of human capital. This implies, in particular, that all full orphans in period $t - 1$ were fairly well educated, in the sense that $e_{t-1}(4)$ is sufficiently close to unity. It also implies that $\bar{\tau}_t(s_t, \cdot) \geq 0$ for all $s_t \neq 4$, with a strict inequality holding in at least one case.

As in section 3.2.1, there is a discrete distribution of the levels of individual adults' human capital, the vector of which is denoted by $[\lambda_t^1, \dots, \lambda_t^{m(t)}]$, where the elements are arranged in

ascending order and, in view of assumption 13, $\lambda_t^1 = \Lambda^a(2, \kappa_{t+1})$. The vector of corresponding measures, normalized by the population of individuals who have just reached adulthood, is denoted by $[\mu_t^1, \dots, \mu_t^{m(t)}]$, with representative element μ_t^k , where μ_t^1 is at least $\pi(\eta_{t-1}, 4)$. Since orphans are not taxable, the total revenue that can be raised in period t without violating $e_t^0(\Lambda_t(s_t), \tau_t(s_t), s_t, \kappa_{t+1}) = 1 \ \forall s_t \neq 4$ is

$$B_t = \sum_{s_t=1}^3 \sum_{k=1}^{m(t)} \bar{\tau}_t(\Lambda_t^k(s_t), \cdot) \cdot \mu_t^k \cdot \pi_t(\eta_t, s_t) N_t. \quad (36)$$

Turning to public expenditures, ensuring universal education cannot be accomplished without a satisfactory system for raising orphans, a system whose exact nature need not detain us here. For each orphan, let the cost of such care and education corresponding to assumption 13 be denoted by $g(4)$. For simplicity, this cost is assumed to be independent of time. Let the proportion $\delta_t(4) \in [0, 1]$ of all full orphans be educated so as to reach at least $\Lambda^a(2, \kappa_{t+1})$ as adults. Then the government's budget constraint is written as

$$B_t - \left[\delta_t(4) \cdot \sum_{k=1}^{m(t)} \mu_t^k \cdot \pi_t(\eta_t, 4) \cdot g(4) + \eta_t \right] N_t \geq 0. \quad (37)$$

If, for any given κ_{t+1} , condition (36) is satisfied for $\delta_t(4) = 1$ and $\eta_t = 0$, then a policy of full education for all is certainly feasible in period t , even when nothing is done to stem the epidemic in that period.

This brings us to optimum policy. Define the residual revenue after financing the care and education of orphans, given expectations regarding η_{t+1} and hence κ_{t+1} as:

$$Q_t(\eta_t, \eta_{t+1}, \delta_t, \cdot) \equiv B_t - \delta_t(4) \cdot \sum_{k=1}^{m(t)} \mu_t^k \cdot \pi_t(\eta_t, 4) \cdot g(4) \cdot N_t,$$

or, since $\sum_{s_t=1}^4 \pi_t(\eta_t; s_t) = 1$,

$$Q_t(\eta_t, \eta_{t+1}, \delta_t, s_t, \kappa_{t+1}) \equiv \left[\sum_{s_t=1}^3 \sum_{k=1}^{m(t)} [\bar{\tau}_t(\Lambda_t^k(s_t), \cdot) + \delta_t(4)g(4)] \cdot \mu_t^k \cdot \pi_t(\eta_t, s_t) - \delta_t(4)g(4) \right] N_t. \quad (38)$$

Differentiating with respect to η_t , we have

$$\frac{\partial Q_t}{\partial \eta_t} = \sum_{s_t=1}^3 \sum_{k=1}^{m(t)} [\bar{\tau}_t(\Lambda_t^k(s_t), s_t, \kappa_{t+1}) + \delta_t(4)g(4)] \cdot \mu_t^k \cdot \pi'_t(\eta_t, s_t) N_t. \quad (39)$$

Since $\bar{\tau}_t^k(s_t = 1) > \bar{\tau}_t^k(s_t = 2) = \bar{\tau}_t^k(s_t = 3)$, (34) implies that $\partial Q_t / \partial \eta_t > 0$. Hence, even if $Q_t(0, \eta_{t+1}, \cdot) < 0$, there may exist a positive value of η_t that does satisfy (36). Denote the largest value of η_t that satisfies (36) when $\delta_t(4) = 1$ by η_t^0 . Then, by the definition of Q_t , we have the maximum feasible level of spending on health, subject to the constraint that all children enjoy full schooling.

We now prove that the expenditure policy $(\delta_t(4) = 1, \eta_t^0, e_t(s_t) = 1 \forall s_t)$, in conjunction with the tax policy described above, also yields the maximal value of Q_{t+1} when continued in the same fashion in period $t + 1$. First, note that children born into households with human capital $\Lambda_t^k(s_t)$ can attain at most

$$\lambda_{t+1}^k = z(s_t)f(1)\Lambda_t^k(s_t) + 1$$

as adults in period $t + 1$, and that they will actually do so under the said policy. Hence, for any given η_{t+1} , there is no other policy that can yield a higher taxable capacity of any individual entering adulthood at the start of period $t + 1$. The children in question make up the following proportions of all adults at the start of period $t + 1$:

$$[\pi_t(\eta_t, 1), \pi_t(\eta_t, 2), \pi_t(\eta_t, 3), \pi_t(\eta_t, 4)] \cdot \mu_t^k.$$

By maximizing η_t , it is clear from $\bar{\tau}_t(s_t = 1) > \bar{\tau}_t(s_t = 2) = \bar{\tau}_t(s_t = 3)$ and assumption 18 that the resulting proportions maximize Q_{t+1} for any given value of η_{t+1} .

The above results may be summarized as:

Lemma 6

If all adults at the start of period t possess at least $\Lambda^a(2, \kappa_{t+1})$ units of human capital and some possess strictly more, and if η_t^0 is positive, then for any given κ_{t+1} , the expenditure cum tax policy in period t described by the vector

$$\Gamma_t = [\eta_t^0, \delta_t(4) = 1, e(\Lambda_t(s_t), \frac{\bar{\tau}_t(\Lambda_t^k(s_t), \cdot)}{\alpha}, s_t, \kappa_{t+1}) = 1 \bar{\tau}_t(\Lambda_t^k(s_t), \cdot) \forall s_t, \bar{\tau}_t(\Lambda_t^k(s_t), \cdot)]$$

is optimal in the following sense:

1. Aggregate human capital at the start of period $t + 1$ is maximized.
2. All adults attain at least $\Lambda^a(2, \kappa_{t+1})$ at the start of period $t + 1$.
3. Q_{t+1} is maximal for any given κ_{t+1} .

For the remainder of this section, we confine ourselves to settings in which there are no further changes in the disease environment after the outbreak of the epidemic at the start of period zero. Let $D \in \{0, 1\}$ denote whether the onset of the epidemic has occurred. Expressed precisely, we employ

Assumption 19. $\pi_t(\cdot) = \pi(\eta_t; s_t, D)$, where $D = 0$ up to period 0, and $D = 1$ thereafter.

It should be noted that the level of premature adult mortality still depends on public policy through the spending on measures to combat the disease once it has broken out. With the help of this assumption, we obtain the following result:

Proposition 8

If the conditions for the validity of lemma 6 hold at time t , and if $z(2)f(1) \geq 1$, then

- (i) the policy program described by the sequence $\{\Gamma_t, \Gamma_{t+1}, \dots\}$ with $\eta_{t+t'} = \eta_t^0 \forall t' \geq 0$ is feasible, and
- (ii) it is dominated by the feasible sequence in which η is maximized in each period with the static expectation that $\eta_{t+t'+1} = \eta_{t+t'} \forall t' \geq 0$.

Proof :

Consider the set of all adults possessing λ_t^k at the start of period t . Given η_t , the measures of the subsets comprising the tax base are $[\pi(\eta_t, 1), \pi(\eta_t, 2), \pi(\eta_t, 3)]\mu_t^k$. The children born to the adults possessing λ_t^k at the start of period t will be sorted into the following groups of adults at the start of period $t + 1$:

- $\pi(\eta_t, 1)$ with human capital $2z(1)f(1)\lambda_t^k + 1$,
- $(\pi(\eta_t, 2) + \pi(\eta_t, 3))$ with human capital $z(2)f(1)\lambda_t^k + 1$,
- $\pi(\eta_t, 4)$ with human capital $\Lambda^a(2, \kappa_{t+1})$.

Recalling assumption 2, it is seen that if $z(2)f(1) \geq 1$, then the human capital of each of those adults who were not full orphans in period t will exceed his or her parents' level. Since those who grew up as full orphans cannot be taxed if their children are to have full-time schooling in period $t + 1$, the total taxable capacity of the above groups in period $t + 1$, defined in relation to the policy in lemma 6, exceeds their parents' level thereof in period t if the following two conditions are satisfied: first, that the proportion of full orphans be no larger in period $t + 1$ than in period t – equivalently, that $\eta_{t+1} \geq \eta_t$; and second, that $\eta_{t+2} \geq \eta_{t+1}$. Suppose, therefore, that the government chooses $\eta_{t+1} = \eta_t^0$. This choice certainly satisfies $\frac{1}{N_{t+1}}Q_{t+1}(\eta_t^0, \eta_t^0, \cdot) > \eta_{t+1} = \eta_t^0$, so that the sequence $\{\Gamma_t, \Gamma_{t+1}, \dots\}$ with $\eta_{t+t'} = \eta_t^0 \quad \forall t' \geq 0$ is a feasible policy program. In view of what it accomplishes, namely, full education for all and the elimination of all danger that any lineage will slide into the poverty trap, it also deserves to be called a 'fairly good' program.

It can be improved upon, however. For by making η_{t+1} a little larger than η_t^0 , the condition $\frac{1}{N_{t+1}}Q_{t+1}(\eta_{t+1}, \cdot) \geq \eta_{t+1}$ will not be violated, there being some taxable capacity to spare, even without taking into consideration the accompanying reduction in the burden imposed by orphans, as expressed by $\pi(\eta_{t+1}, 4)$. This adjustment will also increase not only aggregate human capital at the start of period $t + 2$, but also taxable capacity in that period. The same

reasoning applies to all future periods. ■

In the light of section 3.4, it should be noted the credibility of the policy $\eta_{t+t'} = \eta_t^0 \forall t' \geq 0$ is an essential element of a ‘good’ policy program.

The fact that Q_{t+1} is maximal given $\kappa_{t+2} = \kappa_{t+1}$ suggests that the argument used to prove proposition 4 can also be used to establish sufficient conditions for sustainable growth in the present setting.

Proposition 9

Suppose $z(2)f(1) \geq 1$. Then, starting from the initial conditions described in lemma 6, a sufficient condition for unbounded growth under the policy program $\{\Gamma_t, \Gamma_{t+1}, \dots\}$ with $\eta_{t+t'} = \eta_t^0 \forall t' \geq 0$ is

$$[\pi(\eta_t^0, 1) \cdot 2z(1) + (\pi(\eta_t^0, 2) + \pi(\eta_t^0, 3))z(2)]f(1) \geq 1. \quad (40)$$

Proof :

Note from (34) that taxable capacity as defined in the policy Γ_t in lemma 6 is linear in the aggregate human capital of the individuals possessing more than $\Lambda^a(2, \kappa_{t+1})$. With reference to the group of adults possessing λ_t^k in period t , the tax base provided by their offspring in period $t + 1$ comprises a diverse group of families with the following measures: first, there are the children of two-parent families in period t , whose unions form the vector

$$[\pi(\eta_{t+1}, 1), \pi(\eta_{t+1}, 2), \pi(\eta_{t+1}, 3)]\pi(\eta_t, 1)\mu_t^k N_{t+1},$$

with each adult possessing human capital $2z(1)f(1)\lambda_t^k + 1$, and second, there are the children of one-parent families in period t , whose unions form the vector

$$[\pi(\eta_{t+1}, 1), \pi(\eta_{t+1}, 2), \pi(\eta_{t+1}, 3)](\pi(\eta_t, 2) + \pi(\eta_t, 3))\mu_t^k N_{t+1},$$

with each adult possessing human capital $z(2)f(1)\lambda_t^k + 1$. The aggregate human capital in

the tax base is, therefore,

$$(1 + \pi(\eta_t, 1) - \pi(\eta_{t+1}, 4)) \{ [\pi(\eta_t, 1) \cdot 2z(1)f(1) + (\pi(\eta_t, 2) + \pi(\eta_t, 3))z(2)f(1)]\lambda_t^k + [1 - \pi(\eta_t, 4)]\} \mu_t^k N_{t+1},$$

which exceeds that in period t , namely, $[1 + \pi(\eta_t, 1) - \pi(\eta_t, 4)]\lambda_t^k \mu_t^k N_t$, if and only if

$$\begin{aligned} (1 + \pi(\eta_t, 1) - \pi(\eta_{t+1}, 4)) \{ [\pi(\eta_t, 1) \cdot 2z(1)f(1) + (\pi(\eta_t, 2) + \pi(\eta_t, 3))z(2)f(1)]\lambda_t^k \\ + [1 + \pi(\eta_t, 1) - \pi(\eta_t, 4)] \} N_{t+1} \\ > [1 + \pi(\eta_t, 1) - \pi(\eta_t, 4)]\lambda_t^k N_t. \end{aligned}$$

Now, it has been established in proposition 8 that if $z(2)f(1) \geq 1$, then there is a feasible sequence of policies such that $\eta_{t+1} \geq \eta_t$. It follows that a sufficient condition for aggregate human capital to grow faster than the population of adults in period t is

$$[\pi(\eta_t^0, 1) \cdot 2z(1)f(1) + (\pi(\eta_t^0, 2) + \pi(\eta_t^0, 3))z(2)f(1)] \geq 1,$$

that is to say, the weighted sum of the ‘growth factors’ of one- and two-parent households is at least unity, where their weights are their respective probabilities of occurrence under the policy Γ_t . The claim then follows at once from assumption 2, (34) and proposition 8. ■

It should be remarked that since the policy program $\{\Gamma_t, \Gamma_{t+1}, \dots\}$ with $\eta_{t+t'} = \eta_t^0 \forall t' \geq 0$ can be improved on by somewhat increasing the said level of spending on measures to combat the disease in period $t + 1$ and thereafter, condition (39), while sufficient, is not a necessary condition for unbounded long-term growth. There is, however, a limit on how much premature adult mortality can be reduced by such spending, which will surely run into sharply diminishing returns well before premature mortality disappears – if that were possible. The government will therefore find good grounds for allowing private consumption to grow at some point in the sequence.

4.1.2 The First Phase

Having characterized a ‘fairly good’ policy program and the associated sufficient conditions for unbounded growth once all children receive full schooling, we turn to the task of analyzing the traverse to this state, starting from period zero, when the society is homogeneous and the epidemic is about to break out. Depending on the level of λ_0 and the expected level of κ_1 , achieving a traverse may entail taxation of one- and two-parent families on such a scale as to rule out full-time schooling for their children. To allow for this possibility, the definition of taxable capacity used above must be discarded. Let the tax on a household of type s_0 possessing $\Lambda_0(s_0)$ be denoted by $\tau_0(\Lambda_0(s_0), s_0)$. If there exists a positive $\tau_0(2\lambda_0, 1)$ that satisfies

$$z(1)f(e_0^0(2\lambda_0, \tau_0(2\lambda_0, 1), 1, \kappa_1))2\lambda_0 + 1 > \Lambda^a(2, \kappa_1), \quad (41)$$

then, despite the outbreak of the disease in period 0, there will be some taxable capacity in both periods 0 and 1, conditional on all the children of two-parent households in period 0 choosing full-time education for their own children in period 1. Put formally, we have

Assumption 20. For any κ_1 not too close to zero, λ_0 is large enough to satisfy condition (40).

The government’s budget constraint in period 0 takes the simple form

$$[\pi(\eta_0, 1)\tau_0(2\lambda_0, 1) + (\pi(\eta_0, 2) + \pi(\eta_0, 3))\tau_0(\lambda_0, 2)] - \delta_0(4)\pi(\eta_0, 4)g(4) - \eta_0 \geq 0. \quad (42)$$

Note that $\tau_0(\lambda_0, 2)$ may be negative, in which case, it denotes a subsidy to all single-parent households. Observe also that under assumption 20, (41) can certainly be satisfied by the policy $\tau_0(\lambda_0, 2) = \delta_0(4) = 0$ and $\eta_0 = 0$.

We proceed by forward induction. Suppose there exist a non-negative η_0 and a tax vector $(\tau_0(2\lambda_0, 1), \tau_0(\lambda_0, 2))$ that satisfy (41) and the condition $\Lambda_1(s_1) \geq \Lambda^a(2, \kappa_2) \forall s_1$ when $\delta_0(4) = 1$. Recall that η_t^0 is the largest value of η_t that satisfies (36) when $\delta_t(4) = 1$. If $\eta_t^0 > 0$, some families must be paying taxes, and lemma 6 and propositions 8 and 9 will

apply from period 1 onwards. It is clear that the best chance of achieving the traverse in one period is to chose a program of taxes and subsidies in periods 0 and 1 so as to make Q_1 as large as possible. To get around the difficulty that Q_1 depends on η_2 , we impose the condition that $\eta_2 = \eta_1$, thereby bringing proposition 8 into play, and so ensuring the sustainability of the process after the traverse has been completed. Stated formally, the problem is

$$\max_{[\eta_0, \eta_1, \tau_0(1), \tau_0(2)]} Q_1(\eta_1, \eta_2, \delta_1(4), \cdot) \quad (43)$$

subject to: (41), $\eta_0 \geq 0$, $\eta_1 \geq 0$, $\eta_2 = \eta_1$, $\delta_0(4) = \delta_1(4) = 1$ and $\Lambda_1(s_1) \geq \Lambda^a(2, \kappa_1) \forall s_1$.

If the value of Q_1 at the optimum is positive, the traverse can indeed be accomplished in one period. Otherwise, some full orphans will go uneducated in period 0, thereby enlarging the pool of children to be supported in period 1. That being so, the aim is to show that one can find a program that eventually yields a setting in which lemma 6 does apply.

The principles underlying the solution will be clear from an examination of the case where the traverse can be completed in just two periods. By choosing not to educate all orphans in period 0, the government creates a new need in period 1, namely, to support the children of the uneducated adults who do not die prematurely, where the latter can be thought of as the ‘backlog’ from period 0. Denote the required subsidy per child by $g(s_1)$, to allow for the possibility that this will vary according to the state of the family. Then the aggregate subsidy that must be financed in period 1 is

$$\left[\pi(\eta_1, 4)g(4) + (1 - \delta_0(4))\pi(\eta_0, 4) \sum_{s_1=1}^3 \pi(\eta_1, s_1)g(s_1) \right] N_1.$$

In order to calculate the total revenue collected in period 1, recall that the children of two- and one-parent families in period 0 will attain the following levels of human capital as adults in period 1:

$$\lambda_1^1 = z(1)f(e_0^0(2\lambda_0, \tau_0(2\lambda_0, 1), 1, \kappa_1))2\lambda_0 + 1 \quad (44)$$

and

$$\lambda_1^3 = \lambda_1^2 = z(2)f(e_0^0(\lambda_0, \tau_0(\lambda_0, 2), 2, \kappa_1))\lambda_0 + 1, \quad (45)$$

respectively. Note that λ_1^k in this case is simply $\lambda_1^{s_0}$ and we use the latter notation in this subsection. Recalling that those ‘needy’ children who were supported in period 0 attain $\Lambda^a(2, \kappa_2)$, the total revenue so collected may be written as a special case of the general budget constraint in (25)

$$B_1 = \sum_{s_1=1}^3 \left[\left(\sum_{s_0=1}^3 \tau_1(\Lambda_{s_0}^k(s_1), s_1) \pi(\eta_1, s_1) \pi(\eta_0, s_0) \right) + \delta_0(4) \tau_1(\Lambda^a(2, \kappa_2), s_1) \pi(\eta_1, s_1) \pi(\eta_0, 4) \right] N_1 \quad (46)$$

Hence, the net fiscal resources available for spending on combating the disease, conditional on all children being so supported, are

$$\tilde{Q}_1 \equiv B_1 - \left[\pi(\eta_1, 4)g(4) + (1 - \delta_0(4))\pi(\eta_0, 4) \sum_{s_1=1}^3 \pi(\eta_1, s_1)g(s_1) \right] N_1. \quad (47)$$

The policy problem therefore involves the following modification of problem (42):

$$\max_{[\eta_0, \eta_1, \delta_0(4), \tau_0(s_0), \tau_1(s_1)]} [\tilde{Q}_1] \quad (48)$$

subject to (41), $\eta_0 \geq 0, \eta_1 \geq 0, \delta_0(4) \in [0, 1], \delta_1(4) = 1$ and $\Lambda_2(s_2) \geq \Lambda^a(2, \kappa_2) \forall s_2$.

Let $\text{argmax } \tilde{Q}_1 = [\eta_0^*, \eta_1^*, \delta_0^*(4), \tau_0^*(s_0), \tau_1^*(s_1)]$. If \tilde{Q}_1 is positive at the optimum, then the program $[\eta_0^*, \eta_1^*, \delta_0^*(4), \tau_0^*(s_0), \tau_1^*(s_1)]$ will yield a setting where lemma 6 and propositions 8 and 9 are applicable.

We now investigate problem (47) in some detail, eschewing the Lagrange method in favor of an approach that yields more intuition about the trade-off between spending on education in the form of subsidies and combating the disease in period 0 in the light of their contribution to completing the transition to the desired setting in period 1. Since condition (41) will surely bind at the optimum and there is some taxable capacity when $\eta_0 = 0$, we investigate what

will happen if $\delta_0(4)$ is increased slightly at the expense of η_0 . Such a shift will yield more taxpayers in period 1 and fewer offspring of uneducated parents to subsidize in period 1. The reallocation yields the following change in revenue \tilde{Q}_1 in period 1:

$$\left[\sum_{s_1=1}^3 \left(\tau_1(\Lambda^a(2, \kappa_2), s_1) \pi(\eta_0, 4) \pi(\eta_1, s_1) + \pi(\eta_0, 4) \pi(\eta_1, s_1) g(s_1) \right) \right] N_1 \cdot d\delta_0(4)$$

The drawback of this reallocation is that it will adversely affect the composition of the population in period 1 where taxable capacity is concerned. The corresponding change in revenue induced by a change in η_0 is

$$N_1 \cdot \left[\sum_{s_1=1}^3 \left(\sum_{s_0=1}^3 \tau_1(\Lambda_1^{s_0}(s_1), s_1) \pi'(\eta_0, s_0) \pi(\eta_1, s_1) \right) + \delta_0(4) \tau_1(\Lambda^a(2, \kappa_2), s_1) \pi'(\eta_0, 4) \pi(\eta_1, s_1) - (1 - \delta_0(4)) \pi'(\eta_0, 4) \pi(\eta_1, s_1) \cdot g(s_1) \right] d\eta_0,$$

where it should be recalled from assumption 18 that $\pi'(\eta_0, 1) > 0$ and $\pi'(\eta_0, s_0) < 0$ for $s = 2, 3, 4$. The last term in parentheses expresses the fact that reduced spending on health in period 0 entails more full orphans in period 0, and hence, if $\delta_0(4)$ stays unchanged, more children of uneducated parents to subsidize in period 1.

The final step is to establish how η_0 responds to $\delta_0(4)$ at the margin, a relationship we establish by differentiating (41) totally. Since (41) holds as an equality at the optimum, and $\sum_{s_t=1}^4 \pi(\eta_t, s_t) = 1$, we have

$$\left[\left(\sum_{s_0=1}^3 (\tau_0(\Lambda_0(s_0), s_0) + g(4) \delta_0(4)) \pi'(\eta_0, s_0) \right) - 1 \right] d\eta_0 - \left[\pi(\eta_0, 4) \cdot g(4) \right] \cdot d\delta_0(4) = 0,$$

or

$$\frac{d\delta_0(4)}{d\eta_0} = \frac{[\sum_{s_0=1}^3 (\tau_0(\Lambda_0(s_0), s_0) + g(4) \delta_0(4)) \pi'(\eta_0, s_0)] - 1}{\pi(\eta_0, 4) \cdot g(4)}. \quad (49)$$

Since $\delta_0(4)$ can be increased only at the expense of η_0 when the traverse lasts more than one period, it follows that $(\eta_0^*, \delta_0^*(4))$ must satisfy

$$1 > \sum_{s_0=1}^3 \left\{ \tau_0(\Lambda_0(s_0), s_0) + g(4) \delta_0^*(4) \right\} \pi'(\eta_0^*, s_0). \quad (50)$$

Condition (49) states that a (small) unit increase in spending on health in period 0 must be strictly greater than the combined increase in taxes and the saving of subsidies to orphans that it brings about in that period. For otherwise, the said act of spending would at least finance itself, and so warrant further spending.

To sum up, the functions $\pi(\eta, s)$, ($s = 1, 2, 3, 4$) represent the effectiveness of spending on combating the causes of premature mortality among adults. If the derivatives $\pi'(0, s_0)$ are sufficiently large in absolute magnitude, it will always be optimal to spend something on such measures at the very outset, even at the expense of educating fewer orphans and dealing with the ‘backlog’ later.

4.2 Pooling

We confine our attention to the comparatively simple case of complete pooling, so that we can use the representative family approach developed in section 2.3. Since all children are raised and treated identically, targeted subsidies to promote education directly, which are effective in a nuclear family setting, will be neutralized through reallocations within the family sphere itself. As long as the social institution of pooling remains intact, all individuals within a generation will be identical, and the government need choose only a poll tax on surviving adults, say τ_t for each ‘pair’, in order to raise revenue. Since the measure of all ‘pairs’ at the start of period t is N_t and the measure of tax-paying units is $\kappa_t N_t$, the government’s budget constraint in that period is:

$$\kappa_t N_t \tau_t \geq \eta_t N_t,$$

whereby, in a system of pooling, all individuals will be treated in the same way. This is so by definition when expenditures produce public goods, such as mass awareness campaigns; but it is also a feature of this social arrangement where individual treatment, such as the allocation of drugs, is concerned. In the absence of subsidies of any kind, the budget constraint will hold as an equality, so that $\kappa(\eta_t) = \kappa(\kappa_t \tau_t)$. By virtue of assumption 18 and $\kappa(0) > 0$,

the implicit function theorem yields a unique, increasing function $\kappa(\tau_t)$ ⁸. In what follows, we assume that $\kappa'(0)$ is large, as seems plausible in the case of communicable diseases.

Whether such a society with a constant level of n_t can withstand the onset of an epidemic depends on whether the human capital attained by the children can be held to at least the level of their parents' generation through the use of health policy alone. The evolution of human capital is given by:

$$\lambda_{t+1} = 2z(0, \kappa_t(\tau_t))f(e_t^0(2\lambda_t, \tau_t, 0, \kappa_{t+1}))\lambda_t + 1 \quad (51)$$

where λ_t is the human capital of an adult, and $(c_t^0(0), e_t^0(0))$ maximize $EU_t(0)$ subject to

$$[2 + (\eta_t/\kappa_t)\beta]c_t(0) + \alpha(\eta_t/\kappa_t)\gamma e_t(0) = \alpha[2\lambda_t + (\eta_t/\kappa_t)\gamma] - \tau_t.$$

Note that $e_t^0(0)$ depends on κ_t as well as κ_{t+1} , which makes the design of optimal policies a non-trivial exercise. Let

$$\tau^*(\lambda_t) = \operatorname{argmax}_{\tau_t \in (0, \infty)} \left\{ 2z(0, \kappa_t(\tau_t))f(e_t^0(2\lambda_t, \tau_t, 0, \kappa_t(\tau_t)))\lambda_t + 1 \right\}$$

A sufficiently large $\kappa'(0)$ ensures that $\tau^*(\lambda_t)$ exists. Note that the definition of $\tau^*(\lambda_t)$ assumes that $\kappa_{t+1} = \kappa_t$. If

$$e_t^0(2\lambda_t, \tau^*(\lambda_t), 0, \kappa_t(\tau^*(\lambda_t))) = 1,$$

then $\tau^*(\lambda_t)$ is monotonically increasing in λ_t . For $e_t^0(0) < 1$, however, the monotonicity of $\tau^*(\lambda_t)$ with respect to λ_t is not assured without further assumptions regarding the shape of the functions f and $e_t^0(0)$. We obtain:

Proposition 10

Suppose a pooled society with human capital λ_0 per adult is assailed by the disease at $t = 0$.

If

$$2z(0, \kappa(\tau^*(\lambda_0)))f(e_0^0(2\lambda_0, \tau^*(\lambda_0), 0, \kappa(\tau^*(\lambda_0))))\lambda_0 + 1 > \lambda_0, \quad (52)$$

⁸Note that our formulation implies a simultaneous effect, since healthy adults can be taxed, and tax revenues, in turn, affect the mortality rate. The problem can be avoided without affecting our results by a sequential dynamic approach also within periods, as discussed in section 3.1.3.

there exists a sequence of tax and health policies $\{\tau_t, \eta_t\}_{t=0}^{\infty}$ and an associated time path $\{\lambda_t, \kappa_t\}_{t=0}^{\infty}$ such that a collapse is averted. There will be self-sustaining growth if

$$2z(0, \kappa(\tau^*(\lambda_0)))f(1) \geq 1.$$

Proof of proposition 10:

Simply consider the policy scheme

$$\{\tau_t, \eta_t\}_{t=0}^{\infty} \text{ with } \tau_t = \tau^*(\lambda_0) \text{ and } \eta_t = \kappa(\tau^*(\lambda_0)) \cdot \tau^*(\lambda_0) \equiv \bar{\eta} \quad \forall t$$

We assume for the moment that this scheme is feasible. We will then show that this is indeed so. Note that η_t is constant under this policy regime, for the tax levied is constant over all periods. Hence, $\kappa_t = \bar{\kappa} = \kappa(\bar{\eta})$, and the mortality profile is time-independent as well.

Suppose that adults expect that $\kappa_t = \bar{\kappa}$ in all periods. If condition (52) holds, they will choose $e_0^0(0)$ such that under the policy $\tau^*(\lambda_0)$ and $\bar{\kappa}$, $\lambda_1 > \lambda_0$. To finance $\eta_1 = \bar{\eta}$ in period 1, the government again levies $\tau^*(\lambda_0)$, which is feasible, since $\lambda_1 > \lambda_0$. If adults now expect that $\kappa_2 = \bar{\kappa}$, then

$$\begin{aligned} \lambda_2 &= 2z(0, \bar{\kappa})f\left(e_1^0\left(2\lambda_1, \tau^*(\lambda_0), 0, \bar{\kappa}\right)\right)\lambda_1 + 1 \\ &> 2z(0, \bar{\kappa})f\left(e_0^0\left(2\lambda_0, \tau^*(\lambda_0), 0, \bar{\kappa}\right)\right)\lambda_0 + 1 \\ &= \lambda_1 > \lambda_0 \end{aligned}$$

Under expectations $\kappa_t = \bar{\kappa} \forall t$, the argument can be applied repeatedly. Accordingly, in every period, normalized health expenditures in the amount of $\bar{\eta}$ ($\bar{\eta}N_t$ in aggregate terms) can be financed, and given the policy $\{\tau_t = \tau^*(\lambda_0), \eta_t = \bar{\eta}\}$, the expectations of adults will be correct. Therefore, there exists a time path under rational expectations such that a collapse is averted.

Given this policy, it follows at once that there will be self-sustaining growth if the growth factor $2z(0, \kappa(\tau^*(\lambda_0))) \cdot f(1)$ is at least unity. ■

Several remarks are in order. First, the policy $\tau_t = \tau^*(\lambda_0)$ is not, in general, optimal. If the goal is to maximize the long-term rate of growth of productivity, it will be necessary to increase spending on health as λ_t rises in order to increase the transmission factor $z(0, \kappa_t)$. The right sequence of policies depends, of course, on exactly how the objectives are stated. For instance, if the aim is to maximize long-term aggregate output (and thus aggregate human capital), the optimal policy will involve minimizing the time needed to attain $e^0(0) = 1$, and subsequently maximizing κ_t in each period, subject to ensuring full education for all children.

Second, note once more that expectations about future health policy, and thus about the children's survival prospects as adults, play an essential role in determining the effectiveness of policies today. Expectations that mortality will be higher, and taxable capacity lower, in the future, which in turn imply less generous health policies in the future, can be self-fulfilling, even if health policies today are generous.

Third, the conditions stated in proposition 10 are sufficient for its validity, but in view of the possibility of multiple time paths, they may not be necessary.

In the next step, we provide the counterpart to proposition 10 when a collapse is unavoidable and pooling is a drawback.

Proposition 11

Suppose that a pooled society with human capital λ_0 per adult is assailed by the disease at $t = 0$. If for all $\tau_0 \geq 0$

$$2z(0, \kappa(\tau_0)) f\left(e_0^0(2\lambda_0, \tau_0, 0, 1)\right) \lambda_0 + 1 < \lambda_0,$$

then under any feasible tax and health policy scheme, the economy will collapse.

Proof :

The proof is straightforward. Even if agents expected health policy to be extremely successful in $t = 1$ (namely, $\kappa_1 = 1$), human capital would decline for any health policy chosen in

$t = 0$. Repeating the same observation for subsequent periods yields the assertion. ■

The overall conclusion from this section is that pooling puts the society on a ‘make or break’ road. Since pooling rules out the promotion of education through subsidies targeted at particular subgroups, it can lead to a collapse, which might otherwise be avoidable, especially if the disease causes quite severe mortality. In a less lethal disease environment, in contrast, pooling is a form of social organization that reinforces measures to combat the disease, and so helps fend off the collapse that would occur under a nuclear family structure. On the ‘make road’, the credibility of future measures to ensure low adult mortality is again of decisive importance.

5 An Application to South Africa

5.1 Calibrating the Model

We begin with the fundamental difference equation (3), for which we need the parameters $z(s)$, the functional form $f(e)$ and the boundary value of λ . In view of the highly non-linear nature of the system and the limited information available at this stage of the research, we choose the form $f(e) = e$. Not only does that bring a welcome simplification, but it is also virtually unavoidable if estimation is to proceed. Two further remarks: since the unit time period of the model is a generation, some care is needed when translating the annual flows reported in the available series into parameter values. With this aim in mind, and with two overlapping generations, it is defensible to set the span of each at 30 years. Given that the data are aggregate in form, we are also driven to the heroic assumption that the population is otherwise homogeneous in the period leading up to the full-scale outbreak of the epidemic.

Inspection of the series for GDP reveals that the period from 1960 to 1975 was one of fairly steady and appreciable growth (see Figure 2). By this yardstick, performance thereafter has been at best mediocre. Among the contributing factors were the two oil shocks, the sanctions against the *Apartheid* regime and the restructuring following its collapse. We therefore concentrate on the early sub-period, viewing it as plausible initial basis for assessing how the post-*Apartheid* economy ought to be able to perform over the long haul. Denoting calendar years by the subscript k , and ignoring child labor, we obtain GDP in year k from (5) as

$$Y_k = \alpha L_k \lambda_k, \quad (53)$$

where L_k and λ_k denote the size of the labor force and the average level of efficiency in that year, respectively. Since the labor force series begins in 1965, that year is the initial starting point for the calibration procedure.

Table 2: Values of Y_k , L_k and e_k^B

Year	$Y_k(US\$1995)$	e_k^B	L_k	Y_k/L_k
1960	$49.2 \cdot 10^9$	0.406	n.a.	n.a.
1965	$68.4 \cdot 10^9$	0.410	$7.42 \cdot 10^6$	$9.22 \cdot 10^3$
1970	$90.6 \cdot 10^9$	0.447	$8.24 \cdot 10^6$	$10.99 \cdot 10^3$
1975	$113.0 \cdot 10^9$	0.453	$9.25 \cdot 10^6$	$12.23 \cdot 10^3$
1980	$127.4 \cdot 10^9$	0.461	$10.34 \cdot 10^6$	$12.32 \cdot 10^3$
1985	$132.4 \cdot 10^9$	0.495	$11.93 \cdot 10^6$	$11.10 \cdot 10^3$
1990	$144.7 \cdot 10^9$	0.500	$13.58 \cdot 10^6$	$10.65 \cdot 10^3$
1995	$151.0 \cdot 10^9$	n.a.	$15.29 \cdot 10^6$	$9.88 \cdot 10^3$

Sources: World Bank (2002) and Barro and Lee (1996)

The next step is to anchor the system to λ_{65} . This can be accomplished by progressive substitution using eq. (3), which involves the unknown parameter z and the series for e_t . Although only quinquennial data on educational attainment are available (Barro and Lee, 1996), this is a small drawback in the present setting; for annual fluctuations are of no real significance. The data take the form of the average years of schooling among the population

aged 25 and older – for example, 4.06 years in 1960. Defining ‘full schooling’ as 10 years (6 to 15 inclusive), this yields an average value of e for those born between 1905 and 1935 of 0.406, which is denoted by e_{60}^B . With 1965 as the starting point, and neglecting premature adult mortality, we have

$$\lambda_{70} = 2\hat{z} e_{65}^B \lambda_{65} + 1 \quad (54)$$

and

$$\lambda_{75} = 2\hat{z} e_{70}^B [2\hat{z} e_{65}^B \lambda_{65} + 1] + 1 \quad (55)$$

where \hat{z} is a suitable re-scaling of the parameter z to reflect the averaging effect of the stock in relation to an interval of five years. Substituting for λ_{70} and λ_{75} in (52), we obtain three equations in \hat{z}, α and λ_{65} . The values of Y_k, L_k and e_k in Table 2 then yield $\hat{z} = 0.538$ and the preliminary values $\alpha = 6930, \lambda_{65} = 1.330$.

The estimate for \hat{z} must now be translated into a 30-year setting, with corresponding adjustments to α and λ_0 . The value of λ_{95} can be obtained recursively from $\lambda_{65} = 1.330$ and $\hat{z} = 0.538$ using the series for e_k^B :

$$\lambda_{95} = 2\hat{z} e_{90}^B [2\hat{z} e_{85}^B [2\hat{z} e_{80}^B [\dots] + 1] + 1] + 1 = 0.0147 \lambda_{65} + 2.068 = 2.088.$$

By definition, e_{90}^B is the average schooling of those born between 1935 and 1965, who correspond reasonably well to the population of individuals of school-age in 1965. Returning to eq. (3), therefore, we have

$$\lambda_{95} = 2z e_{90}^B \lambda_{65} + 1 = 1.33z + 1.$$

Hence, $z = 0.818$.

In estimating the associated values of α and λ_{65} , inspection of Figure 2 suggests that 1995 did not represent a normal year on the economy’s long-term growth path, and that 1990, suitably scaled to allow for the shorter time-span of twenty-five years, would be a better choice. That is, we obtain $\lambda_{95}/\lambda_{65}$ from (52) and Table 2, as follows:

$$\lambda_{95}/\lambda_{65} = (\lambda_{90}/\lambda_{65})^{30/25} = 1.189.$$

Substituting $z = 0.818$ into $\lambda_{95} = 2ze_{90}^B \lambda_{65} + 1$ yields $\lambda_{65} = 2.696$, and $\alpha = 3419$ then follows from $\lambda_{65} = \alpha L_{65} \lambda_{65}$.

The final step is to shift the starting point to 1960. At first sight, this appears to be a nicety, especially as the available data impose a calibration process anchored to 1965. As pointed out in the introduction, however, the AIDS prevalence rate rose from about one per cent in 1990 to just over 20 per cent a decade later. This is a strong argument for choosing 1990 as the date of the outbreak of the epidemic in South Africa, and hence 1960 as the starting point in the chosen 30-year framework. The above interpolation from Table 2 yields $\lambda_{95} = 1.189 \lambda_{65} = 3.205$, which implies that λ grew at an annual rate of 0.58 per cent. This yields, in turn, $\lambda_{60} = \lambda_{65}/(1.0058)^5 = 2.620$.

Some comments on these estimates are now in order. First, the parameter α has the dimension of 1995 US dollars per efficiency unit of labor per year. According to these estimates, therefore, a two-parent household in 1960 with two economically active adults and all the children attending school full-time would have had a family income of $\alpha \lambda_{60}$, or \$17,915. In the event of a complete collapse that left the entire population uneducated, the family's income would be just \$6840 in the absence of child labor.

Second, a related aspect of this part of the calibration is the ratio of the labor force to the total population. According to the World Bank's Development Indicators (2002), this ratio stayed constant, at about 0.37, until 1980, and then began to inch up, reaching 0.39 in 1995. It will now be argued that this squares reasonably well with the model's structure under pooling, in which a representative two-parent family cares for (n_t/κ_t) children. In 1960, the total fertility rate was about four (WDI, 2002) and κ_t was about 0.80 (see below), so that (n_t/κ_t) was about five. The corresponding figures for 1990 were about three, 0.86, and 3.5, respectively. Hence, if adults were to work throughout life and children not at all, as in the model, the ratio of the labor force to the total population would be $2/(2 + 5) = 0.286$ in 1960 and $2/(2 + 3.5) = 0.36$ in 1990. Allowing for the model's unit time period of 30 years, the fact that many individuals started to work at fifteen or so and were recorded as in the labor force,

and that those over 64 accounted for only about three to four per cent of the population, the calibration corresponds plausibly to the model's assumption that both members of a couple are economically active.

Third, the estimate of z yields the value of the intergenerational growth factor when children attend school full-time, namely, $2z = 1.636$. This corresponds to an annual growth rate of productivity of about 1.64 per cent over the long run, which seems rather modest in the light of the East Asian experience, but quite in keeping with South Africa's recent performance. All in all, there are grounds for some confidence in the above method of calibration where z , λ_{60} and α are concerned.

Thus far, the form of social organization has remained conveniently in the background, but now that preferences must be specified, a definite choice is unavoidable. For much of the period in question, South Africa was quite rural, so one can make the case that there was widespread pooling. This will be a salient feature of the benchmark cases to be analyzed below. Let preferences be logarithmic, so that (16) takes the form

$$EU_t = 2[a \ln c_t(0) + n_t(\kappa_{t+1}/\kappa_t) \ln \lambda_{t+1}]. \quad (56)$$

Given that the calibration is anchored to 1960, we need both κ_{60} and households' expectations in 1960 concerning the level of κ_{90} . As noted above, the realized value of κ_{90} was 0.86; but we have been unable to find any estimate of κ_{60} . The great reductions in mortality in those three decades benefited children far more than adults, however, so that it seems defensible to set the ratio of the expected value of κ_{90} to the actual value of κ_{60} equal to unity, and this is our choice. At the time of writing, we have not tracked down any studies based on microeconomic data that might yield a clue as to the value of a , so we are driven to another strong assumption based on the aggregates, namely, that

$$e^0(2\lambda_{60}, g = \tau = 0, s = 0; D = 0) = e_{90}^B = 0.5.$$

That is to say, in 1960, a representative 'couple', being unaware of, and untouched by, AIDS in any way, is assumed to have chosen the average years of schooling attained by the gener-

ation born between 1935 and 1965. This yields the value $a = 33.45$.

In order to complete the array of ‘economic’ parameters, we need estimates of β and γ . Setting β at a round one-half, like the judgement of Solomon, seems unobjectionable. A much lower value of γ is called for: $\gamma = 0.2$ yields a maximal level of annual earnings from a child’s labor of $\alpha\gamma = \$685$, which may be on the high side, but we will stay with it for the time being.

We now turn to the demographic components of the model. Fertility is assumed to be exogenous. The population roughly doubled between 1960 and 1990, so that in keeping with assumption 5 and the generation-span of 30 years, each mother had, on average, four surviving children in that period. Whether AIDS will affect fertility in the future is unclear (some evidence points to a modest decline), but what is certain is that AIDS has already contributed to a marked rise in mortality among the under-fives (Dorrington *et al.*, 2001: 21). Since there is also some evidence that fertility had started to fall by the early 1990s (World Bank, 2002), we assume that each mother will have three surviving children from 1990 onwards.

Much more is known about the effects of AIDS on mortality, whereby our exclusive concern in calibrating the model is with premature mortality among adults. The benchmark case is that where there is no epidemic ($D = 0$), which, in view of the low prevalence rate in 1990, is taken to be the age-specific mortality profile for that year, as set out in Dorrington *et al.* (2001, Figure 13: 29). The second reference case is that where the epidemic has reached maturity ($D = 1$) in the absence of any effective measures to combat it. The corresponding profile is assumed to be Dorrington *et al.*’s forecast for 2010. The only snag at this stage is that these authors use what appears to be the demographer’s definition of premature adult mortality, namely, death before the age of 60, conditional on surviving to the age of 15, the probability of which is denoted by ${}_{45}q_{15}$. This span does not suit our purposes; for although entry into the workforce occurs at 15 or so, childbearing starts around 20, and death before 40 would almost surely deprive most or all the children of the parent’s love and care in the later stages of childhood, if not the earlier ones. What is really needed, therefore, is either

Table 3: Estimated sex-specific, premature mortality rates among adults

Year	$_{45}q_{15}(M)$	$_{45}q_{15}(F)$	$_{20}q_{20}(M)$	$_{20}q_{20}(F)$	$_{30}q_{20}(M)$	$_{30}q_{20}(F)$
1990	0.319	0.215	0.106	0.040	0.182	0.098
2010	0.790	0.790	0.359	0.541	0.616	0.707

Note: $_{45}q_{15}$ for 2010 not given separately by sex in Dorrington *et al.* (2001, Table 5: 25).

$_{20}q_{20}$ or $_{30}q_{20}$, whereby the latter corresponds exactly to the 30-year phase of economically productive adulthood, but may be rather too long for the purposes of defining the states $s = 2, 3, 4$. In what follows, we shall use both, but we choose $_{30}q_{20}$ as our reference case. Estimates of these conditional probabilities have been obtained by using piecewise linear interpolations of the profiles in Figure 13 in *ibid.* and applying the product rule. The resulting estimates in Table 3 make for grim reading.

The next step is to calculate the corresponding state-probabilities $\pi_t(s_t)$, a step that requires an assumption about the incidence of the disease among couples. In view of how the disease is transmitted, one is naturally tempted to assume that the infection of one partner outside the relationship would soon be followed by the infection of the other within it, so that viewed within a unit time-period of thirty years, single-parent households would become a rarity. In fact, the probability of transmission within a union appears to be of the order of ten per cent per annum under the conditions prevailing in East Africa (Marseille, Hofmann and Kahn, 2002), which, when cumulated over the median course of the disease from infection to death of a decade, implies that the probability of the event that both partners become infected, conditional on one of them getting infected outside the relationship, is about 0.65. This is high, but still far removed from infection being perfectly correlated within a union. At the present preliminary stage, we plump for simplicity by assuming that the incidence of the disease within a union is i.i.d., an assumption we plan to relax in future work. The resulting state-probabilities based on $_{20}q_{20}$ and $_{30}q_{20}$ are set out in Table 4, where it should be recalled that they correspond not to the actual years shown, but rather to the steady states associated with each disease environment ($D = 0, 1$).

Table 4: Family state-probabilities corresponding to premature adult mortality rates ($_{30}q_{20}$).

Year	$\pi(1)$		$\pi(2)$		$\pi(3)$		$\pi(4)$	
	$_{20}q_{20}$	$_{30}q_{20}$	$_{20}q_{20}$	$_{30}q_{20}$	$_{20}q_{20}$	$_{30}q_{20}$	$_{20}q_{20}$	$_{30}q_{20}$
1990(D=0)	0.855	0.739	0.101	0.164	0.039	0.080	0.005	0.018
2010(D=0)	0.294	0.112	0.165	0.180	0.347	0.272	0.194	0.436

The appalling dimensions – social, economic and psychological – of the epidemic in its mature phase are plain to envisage. In its absence, it is safe to assert on the basis of $_{30}q_{20}$ that at least three-quarters of all children would grow up enjoying the care, company and support of both natural parents, and fewer than two per cent would suffer the misfortune of becoming full orphans. If the epidemic is left to run unchecked, then on the basis of $_{20}q_{20}$, it will leave about 20 per cent of the generation born from 2010 onwards full orphans, about one-half will lose one parent in childhood, and a mere 30 per cent or so will reach adulthood without experiencing the death of one or both parents. The proportions are even more dramatic if, instead, one takes $_{30}q_{20}$ as the basis. The epidemic will also reverse the usual pattern of excess mortality among fathers – from about twice as high as among mothers to about one-third to a half lower. Given the importance of the mother’s special role in securing the young child’s healthy development, it can be argued that this reversal imparts additional force to the shock.

With the reference cases thus defined and numerically estimated, it remains to establish the relationship between the state-probabilities and spending on combating the disease. In keeping with the above procedure, this is accomplished by choosing a functional form for the relationship between the probability of premature death among adults and the level of aggregate expenditures on combating the disease, and then making the assumption that the incidence of the disease is i.i.d. For simplicity, and erring on the side of optimism, we also assume that such aggregate expenditures produce a pure public good, so that

$$q(D = 1) = q(\eta; D = 1), \quad (57)$$

where $q(\eta; D = 1)$ is to be interpreted as the efficiency frontier of the set of all measures that can be undertaken to reduce q in the presence of the disease.

It must be emphasized that while very little is known about the exact shape of $q(\cdot)$, $q(0; D = 1)$ should yield the estimates in Table 3. A second, plausible, condition is that arbitrarily large spending on combating the epidemic should lead to the restoration of the *status quo ante*, that is, $q(\infty; D = 1) = q(D = 0)$. For reasons that will become clear shortly, it is desirable to choose a functional form that not only possesses an asymptote, but also allows sufficient curvature over some relevant interval of η , so that the natural choice falls on the logistic:

$$q(\eta; D = 1) = d - \frac{1}{a + ce^{-b\eta}}. \quad (58)$$

Hence,

$$q(0; D = 1) = d - \frac{1}{a + c}, \quad (59)$$

and

$$q(\infty; D = 1) = d - \frac{1}{a} = q(D = 0). \quad (60)$$

With four parameters to be estimated, two additional, independent conditions are required. One way of proceeding is to pose the question: what is the marginal effect of efficient spending on q in high- and low-prevalence environments, respectively? That is to say, we need estimates of the derivatives of $q(\eta; D = 1)$ at $\eta = 0$ and some value of η that corresponds to heavy spending, when the scope for exploiting cheap interventions has been exhausted. Matters now become decidedly more speculative. In order to obtain such estimates, we draw on the estimated costs of preventing a case of AIDS or saving a DALY by various methods, as reported in Marseille, Hofmann and Kahn (2002, Table 1). When the prevalence rate is high, the most cost-efficient form of intervention is to target prostitutes for the specific purpose of controlling sexually transmitted diseases and promoting the use of condoms. The associated cost per AIDS-case averted in Kenya is given as US \$8-12. In the nature of things and people, it seems reasonable to infer that this cost recurs annually. Other preventive measures are less cost-effective by a factor of up to ten or more. Marseille, Hofmann and Kahn

put the average cost per DALY of a diverse bundle of such measures at \$12.50, whereby it may be remarked that, for these particular interventions, the assumption that η produces a pure public good does not seem to be wide of the mark. Now, a reduction in q of 0.01 over a span of 30 years yields 0.3 DALYs.⁹ Allowing for the fact that there is ‘substitution’ among diseases, that is, if one does not succumb to AIDS, then there is always the threat of something else, the expenditure of another \$12.50 when η is small will yield a net reduction in $q(D = 1)$ of about $(0.01) \cdot (1/0.3) \cdot (1 - q(D = 0)) = 0.028$. Recalling that η is defined with reference to a population of adults whose measure has been normalized to unity, and rounding up to \$15, we have

$$\frac{dq(0; D = 1)}{d\eta} = -\frac{cb}{(a + c)^2} = -\frac{0.028}{15}. \quad (61)$$

Following the purposive and determined implementation of the full battery of preventive measures, the remaining intervention is to treat the infected. There is neither a cure now, nor the prospect of any for perhaps decades to come. Opportunistic infections can, of course, be treated in the later stages of the disease, and the onset of full-blown AIDS can be delayed for a few years through the controlled use of anti-retroviral therapies. Such measures will do little to reduce q as strictly defined, but by keeping infected individuals healthier and extending life a bit, they will raise lifetime income and improve the parental care enjoyed by children in affected families. In the context of the model, therefore, it seems perfectly defensible to interpret these gains as equivalent to a reduction in q . Marseille, Hofmann and Kahn (2002) put the cost of saving a DALY by such means at \$395, assuming that the drugs take the form of low-cost generics and explicitly neglecting the costs of the technical and human infrastructure needed to support an effective, so-called HAART, regimen of this kind. This estimate must therefore be regarded as an optimistic one where the cost-effectiveness of treatment, as opposed to prevention, is concerned. Be that as it may, it is assumed here that HAART is the efficient, marginal form of intervention when a low prevalence rate has resulted from a determined, extensive and continuing effort at prevention. In order to com-

⁹The procedure for ${}_{20}q_{20}$ is set out in section 5.3.1.

plete the specification of this case, it must be determined at what level of aggregate spending HAART becomes the best choice at the margin. Note that in the absence of diminishing returns to preventive measures, it would be possible to attain the *status quo ante* ($D = 0$) by spending

$$[q(0; D = 1) - q(D = 0)] \cdot (15/0.028) = 278.$$

In fact, diminishing returns will set in as the prevalence rate falls. Where preventing mother-to-child transmission is concerned, for example, a drop in the prevalence rate from 30 to 15 per cent will almost double the cost of saving a DALY (Marseille, Hofmann and Kahn, 2002: 1852). Since 15 per cent hardly counts as a low level of prevalence, it seems fairly safe to assume that HAART will not become cost-efficient until spending on preventive measures and the treatment of opportunistic infections is at least triple the above estimate. Granted this much, we obtain the required fourth condition:

$$\frac{q'(0; D = 1)}{q'(835; D = 1)} = \frac{395}{15}. \quad (62)$$

The four conditions (59) - (5.1) may be solved to yield the values of the parameters a, b, c and d for males, females and both combined, as set out in Table 5. The associated functions $q(\eta; D = 1)$ are plotted in Figure 3.

Table 5: The parameters of the functions $q(\eta; D = 1)$

	a	b	c	d
women	0.6613	0.0051	0.4464	1.6101
men	-0.6555	0.0034	0.1451	-1.3432
average	0.3562	0.4396	0.8145	2.9450

5.2 Benchmarks and Policies: Results

We formulate a set of variants designed to throw light on the efficacy of policy when the overriding aim is to avert an economic collapse following the outbreak of the epidemic.

We begin by describing two benchmark cases, and then proceed to lay out three variations, which are distinguished by the prevailing social structure and the available instruments of intervention. The year 1960 corresponds to $t = -1$, for which an estimate of λ has been derived in section 1 in the form of λ_{60} , and 1990 to $t = 0$, at which point the disease breaks out in all but the first benchmark case.

The common feature of both benchmark cases is ‘pooling’, that is to say, all children are valued and raised in exactly the same way within an extended family structure. Given the number of surviving children born to each mother, changes in premature mortality among adults affect only the number of children raised by each representative ‘couple’. The benchmarks are distinguished solely by the absence or presence of the epidemic, there being no interventions in either case. (The ‘normal’ programs of taxation and public expenditures are already implicitly present in the process of calibration employed in section 5.1). Each generation is therefore completely homogeneous, epidemic or no epidemic. In this connection, we add that we are keenly aware of the inequalities in South Africa, and that the appeal to pooling is a merely convenient device to obtain a simple *status quo ante*. As we have seen, AIDS provides its own powerful impulses towards inequality.

In keeping with the results on pooling in Section 4.2, the tax on each ‘pair’ of adults in period t is chosen so as to maximize a child’s human capital on attaining adulthood at the start of period $t + 1$, all children being treated identically. In constructing the sequence $\{\eta_t, \tau_t\}_{t=0}^{t=T}$, it should be recalled that the condition $\tau_{t-1} \leq \tau_t$ may not be violated; for otherwise, the level of education chosen in period $t - 1$ will have been chosen on the basis of a falsely optimistic expectation about mortality in period t . This consideration turns to be especially important once $e_t^0 = 1$ is attainable, since the effects of a reduction in taxation on the expected mortality factor $\kappa_{t+1}(= \kappa_t)$ may then be outweighed by the resulting increase in λ_{t+1} , with an ensuing jump in mortality from then onwards. All pooling variations therefore involve sequences based on the following problem:

$$\max_{\tau_t} [2z(0, \kappa(\tau_t))f(e_t^0(2\lambda_t, \tau_t, 0, \kappa(\tau_t)))\lambda_t + 1] \quad \text{s.t. } \tau_t \geq 0, \eta_t \geq \eta_{t-1} \quad (63)$$

In order to solve problem (63), the form of $z(0, \kappa(\tau_t))$ must be specified, but very little is known about it. Recalling the discussion in section 2.3.1 that yields assumption 8, we employ the special form

$$z(0, \kappa_t) = \begin{cases} (2z(1) + z(2))\kappa_t/2 & \text{if } \kappa_t < 1/2 \\ (2z(1) - z(2))\kappa_t/2 + z(2)/2 & \text{otherwise} \end{cases} \quad (64)$$

which has a kink at $\kappa_t = 1/2$. Note that in the special case where $z(1) = z(2)/2$, (64) specializes to $z(0, \kappa_t) = z(1)$ for all $\kappa_t \geq 1/2$, which is the form chosen for all the cases analyzed in this sub-section, where it will be recalled from section 5.1 that $z(0, 0.86) = 0.818$.

The three policy variations are distinguished, first, by whether the social institution of pooling survives the increase in child-dependency that results from an increase in premature adult mortality, and second, by whether it is administratively possible to subsidize families conditional on their children attending school, as opposed to supporting them through general lump-sum transfers. If the institution of pooling does withstand the shock, the collection of taxes to finance selective lump-sum transfers will be at best harmless, and at worst positively damaging, through the deadweight losses so induced (though these are not explicitly modeled here); for by definition, the community will fully undo all targeted redistribution intended to bring about inequality. If, however, pooling breaks down, to be superseded by nuclear families, then heterogeneity will emerge, and the targeting of subsidies to promote education will become unavoidable in principle. In all three cases, there is an inherent trade-off at the optimum between spending on measures to combat the disease and promoting the formation of human capital directly by undertaking measures to promote education. Spending on health always takes the form of a public good.

5.2.1 Benchmark 1: pooling, no AIDS

The series $\{\lambda_t, e_t, \eta_t, \kappa_t, n_t, y_t\}_{t=-1}^{t=3}$ are set out in Table 6.1; the corresponding trajectory of the variable λ_t , about which all else revolves, is plotted in Figure 4. The key features of this story are that $\lambda_0 > \lambda^*(0)$ and that steady-state growth is ultimately attained. Starting from the modest level of 0.5 in 1960, education becomes virtually full-time in the generation born from 2020 onwards, by which point, income per head is two-thirds higher than in 1960, with another 80 per cent increase to follow in the next generation. The burden of child-dependency is limited throughout: 0.65 adopted children per ‘couple’ in addition to the four of their own before 1990, and 0.49 in addition to the three thereafter. This is the relatively happy, counterfactual, story into which AIDS intrudes at $t = 0$.

Table 6.1: Benchmark 1: Pooling, no AIDS

year	λ	e	η	κ	n	$y(0)$
1960	2.62	0.50	0	0.86	4.65	19,503
1990	3.14	0.64	0	0.86	3.49	22,335
2020	4.32	0.97	0	0.86	3.49	29,589
2050	7.86	1.00	0	0.86	3.49	53,724
2080	13.85	1.00	0	0.86	3.49	94,715

5.2.2 Benchmark 2: pooling, AIDS and no intervention

The consequences of doing nothing ($\tau = \eta = 0 \forall t \geq 0$) are nothing short of disastrous. The epidemic sets in train a complete collapse of both the economy and, almost surely, the social institution of pooling within a few generations. The extremely high level of premature mortality among adults, as set out in Table 3, leaves the community relatively impoverished from the start and with an intolerable burden of dependency, each surviving ‘couple’ having to care for almost two adopted children for each one of their own. Education is correspondingly neglected, with unrelieved child labor ($e = 0$) for the generation born starting in 2020,

and the descent into backwardness ($\lambda = 1$) is complete by 2050, when family income is a little less than two-thirds its level in 1960, and there are almost twice as many children for each ‘couple’ to care for. Can this fate be averted?

Table 6.2: Benchmark 2: Pooling, AIDS and no intervention

year	λ	e	η	κ	n	$y(0)$
1960	2.62	0.50	0	0.86	4.65	19,503
1990	3.14	0.20	0	0.34	8.87	26,366
2020	2.01	0.00	0	0.34	8.87	17,773
2050	1.00	0.00	0	0.34	8.87	12,901
2080	1.00	0.00	0	0.34	8.87	12,901

5.2.3 Policy option 1: spending on health under pooling

In Benchmark 2, the future of pooling, like everything else, is bleak. It could be, however, that under the optimal program of spending on health, premature mortality among adults will be sufficiently low as to keep pooling viable. We therefore maintain this as an assumption and see what ensues when problem (62) is solved sequentially.

The results are qualitatively striking. The optimal level of spending on combating the epidemic immediately upon outbreak ($t = 0$) is \$963, which corresponds to about 4.5 per cent of GDP, rising to \$1030, or 3.6 per cent of GDP in 2020, when productivity is 30 per cent higher. Fiscally speaking, this is a tall order and a very substantial long-term burden; but if it is politically feasible, it will eventually yield steady-state growth, with full and universal education attained in 2050. With (optimal) spending at this level, premature mortality among adults would be scarcely higher than in the absence of the disease altogether. A comparison with Benchmark 1, as set out in Table 6.1, reveals that the costs of dealing with AIDS in terms of lost output are modest at first, but become quite large by 2080, when productivity is about 88 per cent of its benchmark level, even with the optimal package of interventions under the

favorable conditions of the case considered here. The long-run rate of growth is unaffected by AIDS under this policy program; for once full-time schooling is reached, the growth rate depends only on $z(0, \kappa)$, which is constant, by assumption, at the value $z(0, 0.86) = 0.818$. Taking a somewhat broader view, therefore, the outcome is very encouraging, in that the general qualitative character of Benchmark 1 is still attainable, including a relatively low level of premature adult mortality (see Figure 5). Thus, the maintained assumption that pooling will survive the shock is arguably validated.

Table 6.3: Policy option 1: Spending on combating the disease under pooling

year	λ	e	η	κ	n	$y(0)$
1960	2.62	0.50	0	0.860	4.65	19,503
1990	3.14	0.60	963	0.849	3.53	22,445
2020	4.10	0.87	1029	0.852	3.52	28,365
2050	6.83	1.00	1029	0.852	3.52	46,725
2080	12.18	1.00	1029	0.852	3.52	83,269

5.2.4 Policy option 2: nuclear families, lump-sum subsidies

One problem with the results under pooling option 1 is that they are predicated on the assumption that the government acts at once to nip the epidemic in the bud. In fact, the epidemic had assumed alarming proportions by 2000, with many children already left as orphans and even more destined to become orphans, thus calling into question the whole system of pooling. If the social institution of pooling does break down, leaving tightly defined nuclear families to emerge instead, then the government will be faced with the challenging task, not only of averting a collapse, but also of preserving equality within each generation. In order to make this possible, we need additional assumptions about the formation of human capital when children are left as half- or full orphans. Under the assumption that $z(1) = z(2)/2$, that is, single-parents can do just as well as couples in raising their children if the income is there, it is possible to preserve equality of educational outcomes among all

children with at least one living parent by subsidizing one-parent families so as to induce them to choose the same level of education as that chosen by two-parent families. By hypothesis, no family takes in full orphans, so that these children must be cared for and raised in orphanages. We assume that it is possible for these institutions, when properly staffed and run, to substitute for parents perfectly, at least where the formation of human capital is concerned. The operating rule is that each full orphan will also enjoy the same level of consumption as a child in a single-parent household.

As argued in section 4.1, when the family structure is nuclear, a ‘good’ policy program to overcome the shock caused by AIDS must ensure a substantial tax base, not only in the present, but also in the next generation. The instruments available for this purpose are taxes on two-parent households, spending on combating the disease, the size of the subsidy to single-parent households and the proportions of half- and full orphans to be supported. They are chosen subject to the above restrictions designed to preserve equality, if at all possible, and to the government’s budget constraint.

Given the complexity of using full-scale forwards induction, as set out in section 4.1, we opt here for a somewhat simpler approach, in which the aim is to maximize the expected size of the tax base in the next period, where all parties hold the firm expectation that there will be a continuation of the level of premature adult mortality (and hence of η) prevailing in the present. That is to say, we resort to the assumption that there are stationary expectations, an assumption that permits the maximization problem to be written so that it effectively contains no variables or parameters pertaining to the future. In particular, families’ decisions about education depend on κ_t rather than κ_{t+1} . The (bounded) rationality of these expectations is secured by imposing the condition that η not fall from one period to the next; for this we will rule out policy programs under which the value of investments in education will be reduced *ex post* by failures to take adequate measures against the disease in the next period. It should be emphasized that if it is possible to stave off a collapse of the economy through a policy program derived on the basis of stationary expectations so formulated, then it will certainly

be possible to do still better by using the full apparatus of forwards induction. That, however, is a step we leave to a later stage of the research.

Since all adults possess at least one unit of human capital, the tax base is defined, for present purposes, as the excess of the aggregate level of human capital over unity. By assumption, all adults are identical at the beginning of period 0. If equality is preserved until period t , the policy problem is formulated as follows:

$$\max_{[\eta_t, \tau_t(1), g_t(2), \delta_t(2), \delta_t(4)]} \left\{ \begin{array}{l} \{\pi(\eta_t, 1)[2z(1)e_t^0(2\lambda_t, \tau_t(1), 1, \kappa_t)] \\ + \delta_t(2)(\pi(\eta_t, 2) + \pi(\eta_t, 3))[z(2)e_t^0(\lambda_t, -g_t(2), 2, \kappa_t)] \\ + \delta_t(4)\pi(\eta_t, 4)[z(2)e_t^0(\lambda_t, g_t(2), 2, \kappa_t)]\} \lambda_t \end{array} \right\} \quad (65)$$

subject to:

the instruments' natural intervals, namely,

$$\eta_t \geq 0, \tau_t \geq 0, g_t(2) \geq 0, \delta_t(2) \in [0, 1], \delta_t(4) \in [0, 1], \eta_t \geq \eta_{t-1};$$

the condition that all educated children enjoy the same level of schooling,

$$e_t^0(2\lambda_t, \tau_t(1), 1, \kappa_t) = e_t^0(\lambda_t, -g_t(2), 2, \kappa_t); \quad (66)$$

and the government's budget constraint

$$\begin{aligned} \pi(\eta_t, 1)\tau_t(1) = \eta_t &+ \delta_t(2)(\pi(\eta_t, 2) + \pi(\eta_t, 3))n_t g(2) \\ &+ \delta_t(4)\pi(\eta_t, 4)n_t [\alpha\lambda/H + \beta c_t^0(2) - \alpha\gamma(1 - e_t^0(2))], \end{aligned} \quad (67)$$

where $g_t(2)$ is the subsidy paid to single-parent households and H is the number of orphans each adult can care for in a properly run institution, each such adult receiving the going wage per efficiency unit of labor. It should be remarked that the assumption that $z(1) = z(2)/2$ and condition (66) together permit the maximand to be rewritten as

$$[\pi(\eta_t, 1) + \delta_t(2)(\pi(\eta_t, 2) + \pi(\eta_t, 3)) + \delta_t(4)\pi(\eta_t, 4)][2z(1)e_t^0(2\lambda_t, \tau_t, 1, \kappa_t)\lambda_t],$$

an expression which makes the trade-offs involved somewhat more transparent. If $\delta_t(2) = \delta_t(4) = 1$, then the maximand specializes at the optimum to $[2z(1)e_t^0(2\lambda_t, \tau_t, 1, \kappa_t)\lambda_t]$. By

introducing the constant unity, one obtains the level of human capital attained in period $t + 1$ by any child born in period t , as in problem (62). It should be noted, however, that the form of the government's budget constraint, (67), is not the same as that under pooling. The difference between the two sequences then reveals the advantages of pooling.

It is a matter of great relief that the optimum sequence yields a continuation of growth with complete equality, all orphans receiving the support needed to bring them up to par with the children of two-parent households in each and every period. Growth is distinctly sluggish, however, which points to a collapse that was somewhat narrowly averted. The (uniform) years of schooling rise noticeably more slowly across succeeding generations than under pooling, with full-time schooling achieved only in 2080, when the level of productivity is only slightly more than double its value in 1990. Spending on combating the disease is also higher in absolute terms throughout, and combined with the transfers required to support the needy children, this yields a much heavier fiscal burden than in the pooling case. Two-parent households pay a little over 20 per cent of their income to finance this program in 1990, and find small relief until rapid growth begins from 2080 onwards, and one-parent families need less support.

These differences between policy options 1 and 2 call for some explanation. Recall that under pooling, the objective is to maximize the (uniform) level of efficiency in the next generation, whereas with nuclear families, it is the size of the future tax base that matters when the government has to undertake the task of replacing the institution of pooling with subsidies and orphanages. In the latter case, therefore, it may be worthwhile to trade off educational attainment in order to secure more surviving adults at the later date. That is exactly what has happened here: the absolute level of η is 14 per cent higher than under pooling in both 1990 and 2050, despite the fact that the level of productivity under pooling is 57 per cent higher in the latter period. The other contributing factor arises from the fact that raising children in orphanages draws some adults out of the production of the aggregate private good, a cost that does not arise (by assumption) under pooling. The upshot is that

families have less disposable income than under pooling, so that their children receive fewer years of schooling and growth is much slower. As under pooling, the long-run rate of growth is unaffected by AIDS in this ‘fairly good’ sequence; but the traverse to steady-state growth is a painfully long one.

Table 6.4: Policy option 2: nuclear families, lump-sum subsidies

year	λ	$e(1)$	$e(2)$	η	$g(2)$	τ	$\delta(2)$	$\delta(4)$	κ	$y(1)$
1990	3.14	0.49	0.49	1101	2174	4223	1	1	0.854	22,536
2020	3.51	0.58	0.58	1127	2470	4607	1	1	0.854	24,887
2050	4.36	0.79	0.79	1179	3143	5466	1	1	0.854	30,228
2080	6.60	1.00	1.00	1179	2214	4707	1	1	0.854	45,136

5.2.5 Policy option 3: nuclear families, school-attendance subsidies

The results for this case are qualitatively similar to those under policy option 2, but with the encouraging feature that growth is considerably more rapid. Given the efficiency of school-attendance subsidies relative to lump-sum transfers, and hence the lower taxes on two-parent households, one would expect a swifter attainment of full-time schooling in this variant, and this is indeed the case here. The precise reasoning runs as follows: Choose the optimal levels of taxes on two-parent households and spending on health under policy option 2. This program will yield the same demographic structure, the same level of education among such families, and the same total tax revenues. The outlays under policy option 3 needed to induce the same level of education among the children of one-parent households, however, will be smaller than under policy option 2. These children will also have a lower level of consumption, a standard to which full orphans are tethered. It follows that there will be an excess of total revenue over expenditures. Let η be held constant, so as to keep the demographic structure unchanged, and let the taxes on two-parent households be reduced slightly, which will induce a small rise in $e(1)$. By continuity, there will still be enough funds to finance the additional subsidies to half- and full orphans that will be needed to preserve

equality in education, and hence in human capital in the next generation of adults. It follows that policy option 3 strictly dominates option 2 in all periods from $t = 0$ onwards.

It turns out that full education is reached in 2050, as is the case under pooling, though the level of productivity is 12 per cent lower, due to the accumulated effects of lower attainments in the two preceding generations. Spending on measures to combat the disease is a little higher than under pooling at first, but a little less from 2020 onwards. It is about 13 per cent lower than its counterpart under policy option 2 throughout, so that more premature deaths are implicitly accepted, though the differences in κ_t are small. A measure of the comparative efficiency of conditional educational subsidies is that satisfactory growth is achieved with amounts paid to one-parent households that are barely one tenth of the lump-sum transfers made under policy option 2. The tax burden on two-parent households is correspondingly lighter: the absolute payment per household is a little less than one-half of that under policy option 2 in 1990, rising to 56 per cent in 2080. The difference in productivities is very large at the latter date, namely, $2z(1)$ to one, or 1.636, which implies a much lower relative tax burden. The latter falls from about 8.6 per cent of income in 1990 to 3.6 per cent in 2080 under policy option 3, and from 19.3 per cent to 10.4 per cent under policy option 2.

Table 6.5: Policy option 3: nuclear families, school-attendance subsidies

year	λ	$e(s)$	$e(s)$	η	$g(2)$	τ	$\delta(2)$	$\delta(4)$	κ	$y(1)$
1990	3.14	0.57	0.57	973	280	1886	1	1	0.850	22,337
2020	3.91	0.78	0.78	1022	289	2101	1	1	0.852	27,220
2050	5.99	1.00	1.00	1022	197	2310	1	1	0.852	40,960
2080	10.80	1.00	1.00	1022	0	2642	1	1	0.852	73,839

5.3 Other Variations and Robustness

Some reflection on the results presented in section 5.2 suggests that three aspects of the model's calibration need to be examined in greater detail, with the object of establishing

how robust those findings are to changes in parameter values and specification. These aspects concern the level of premature adult mortality, the dependence of the transmission factor z on family structure and size, and the distinct possibility that the epidemic is well on the way to reaching a mature phase before anything is done to bring it under control. The following sections deal with each in turn, as separate variations on the constellation of parameters that defines the base-case settings in section 5.2.

5.3.1 Mortality

As already indicated in section 5.1, one can argue that ${}_{20}q_{20}$ is a better statistic of premature adult mortality than ${}_{30}q_{20}$ where the effects of such mortality on the prevalence of orphans and the associated burden of caring for them are concerned. Its drawback is that in a structure with an indivisible 30-year period, it leads to a strong under-estimate of the loss of lifetime income arising from mortality among all adults in the economically active age-groups. On balance, the use of ${}_{20}q_{20}$ instead of ${}_{30}q_{20}$ is probably a step away from a conservative stance where estimating the effects of the epidemic are concerned; but some results are reported here.

The estimation of $q(\eta; D = 1)$ proceeds essentially as in section 5.1. The steps involving (59) and (60) are unchanged, but some care is needed with (61), since ${}_{20}q_{20}$ clearly understates the extent of premature adult mortality where the effects on output and income are concerned. We therefore make a suitable adjustment to the number of DALYs saved through a reduction of q . Now, a reduction of 0.01 therein over 20 years yields 0.2 DALYs, as opposed to 0.3 DALYs in connection with ${}_{30}q_{20}$. Hence, (61) is modified to

$$\frac{dq(0; D = 1)}{d\eta} = -\frac{cb}{(a + c)^2} = -\frac{0.0447}{15} \quad (68)$$

for males; the corresponding value for females is $-0.048/15$. The fourth condition involves an intermediate step, which yields values of 85 and 157 for males and females, respectively.

Applying the same argument as in section 5.1, 61 becomes

$$\frac{q'(0; D = 1)}{q'(255; D = 1)} = \frac{395}{15} \quad \text{for males}$$

and

$$\frac{q'(0; D = 1)}{q'(471; D = 1)} = \frac{395}{15} \quad \text{for females.}$$

These four conditions yield the functions $q(\eta)$ graphed in Figure 6.

Despite the drastic reduction in mortality that is entailed in switching from $_{30}q_{20}$ to $_{20}q_{20}$, both the benchmarks and the three policy options yield results that are qualitatively similar to those reported in section 5.2, as can be seen comparing Figures 7 and 8 with their counterparts figures 3 and 4, respectively. In the absence of intervention, there is still a collapse, albeit a slower one, and the long-run costs of the disease are cumulatively large, even with intervention, though less daunting than those under $_{30}q_{20}$.

The other aspect of mortality that needs to be addressed is whether something close to the *status quo ante* can be achieved. In this connection, a striking feature of the results in section 5.1 is that the levels of spending in the policy sequences are so high as to keep the disease almost fully suppressed, as measured by the difference between the statistic κ_t and $\kappa(D = 0)$. This could be regarded as a rather optimistic assessment of the position. A simple alternative is to rule out such a close approach to the *status quo ante* by raising the lower asymptote of the function $q(\eta_t; D = 1)$, so that (59) becomes

$$q(\infty; D = 1) = d - 1/a = Aq(D = 0), \quad A \geq 1.$$

In what follows, we set A equal to 1.1. The corresponding functions $q(\eta_t; D = 1)$ are depicted in Figure 9.

This less tractable disease environment than its counterpart in section 5.2 induces slightly higher spending under all three policy options, as intuition suggests it should. It is seen from Tables 7.1-7.3 that κ falls only a little, from 0.852 to 0.837 under pooling, from 0.855 to 0.842 under policy option 2, and from 0.852 to 0.837 under policy option 3. Yet its effects

on human capital accumulation and output, while very small at first, make themselves felt by 2080. Under pooling, the level of human capital is then just over four per cent lower than its counterpart in section 5.2.3 and 16 per cent lower than in the absence of the disease. The corresponding figures for policy option 2 are much more striking, at 19 and 62 per cent, respectively, and full-time schooling for all is still not quite attained at that date. Those for policy option 3, namely, eight and 27 per cent, lie in between, a further testament to the gains from targeting in this context.

Table 7.1: Variation 1: Benchmark 1: Pooling

year	λ	e	η	κ	n	$y(0)$
1960	2.62	0.50	0	0.860	4.65	19,503
1990	3.14	0.59	973	0.835	3.59	22,492
2020	4.04	0.84	1038	0.837	3.59	28,019
2050	6.53	1.00	1038	0.837	3.59	44,632
2080	11.68	1.00	1038	0.837	3.59	79,845

Table 7.2: Variation 1: Policy option 2: nuclear families, lump-sum subsidies

year	λ	$e(s)$	$e(s)$	η	$g(2)$	τ	$\delta(2)$	$\delta(4)$	κ	$y(1)$
1990	3.14	0.47	0.47	1113	2066	4546	1	1	0.840	22,581
2020	3.40	0.53	0.53	1133	2264	4841	1	1	0.840	24,227
2050	3.96	0.67	0.67	1172	2692	5471	1	1	0.840	27,783
2080	5.32	0.99	0.99	1248	3733	6976	1	1	0.842	36,416

Table 7.3: Variation 1: Policy option 3: nuclear families, school-attendance subsidies

year	λ	$e(1)$	$e(2)$	η	$g(2)$	τ	$\delta(2)$	$\delta(4)$	κ	$y(1)$
1990	3.14	0.55	0.55	980	277	2046	1	1	0.835	22,413
2020	3.82	0.73	0.73	1025	285	2266	1	1	0.837	26,693
2050	5.59	1.00	1.00	1025	247	2250	1	1	0.837	38,237
2080	10.15	1.00	1.00	1025	0	2911	1	1	0.837	69,385

5.3.2 The transmission factor z

Thus far, we have maintained the special assumption that $z(0) = z(1) = z(2)/2$. This, too, is arguably on the optimistic side, so we now allow parents to be complementary, to a degree, in the task of raising their children. Let $z(1) = 0.6z(2)$, that is to say, *ceteris paribus*, two parents are 20 per cent better than one where the formation of human capital is concerned. Since $z(0, 0.86) = 0.818$, the lower branch of (63) yields $z(1) = 0.838$ and $z(2) = 1.396$. Hence, (63) becomes:

$$z(0, \kappa_t) = \begin{cases} 1.537\kappa_t & \text{if } \kappa_t < 1/2 \\ 0.698 + 0.138\kappa_t & \text{otherwise} \end{cases} \quad (69)$$

The effects of replacing $z(0, \kappa_t) = z(1) = z(2)/2 = 0.818$ with the values just derived are as follows. Starting with pooling, the qualitative picture differs not at all from its counterpart in section 5.2, but there are some quantitative differences. In the second benchmark, when nothing is done to combat the disease, the descent into the poverty trap is even swifter than in section 5.2.2 (see Table 8.0); for the burden of caring for so many children now makes itself felt also through a reduction in $z(0, \kappa)$, the upper branch of (69) being in effect. Under policy option 1, the dependence of the transmission factor on κ (and hence η) yields slightly higher spending on health than in section 5.2.3, and the associated increase in taxes exercises a slight dampening effect on the growth of λ (see Table 8.1).

Table 8.0: Variation 2: Benchmark 2: Pooling, AIDS and no intervention

year	λ	e	η	κ	n	$y(0)$
1960	2.92	0.50	0	0.860	4.65	19,503
1990	3.14	0.09	0	0.338	8.67	26,988
2020	1.30	0.00	0	0.338	8.67	14,975
2050	1.00	0.00	0	0.338	8.67	12,901
2080	1.00	0.00	0	0.338	8.67	12,901

Turning to nuclear families and policy options 2 and 3, the replacement of the assumption

Table 8.1: Variation 2, Policy Option 1: Spending on combating the disease under pooling

year	λ	e	η	κ	n	$y(0)$
1960	2.62	0.50	0	0.860	4.65	19,503
1990	3.14	0.60	972	0.850	3.53	22,446
2020	4.09	0.87	1034	0.852	3.52	28,323
2050	6.80	1.00	1034	0.852	3.52	46,511
2080	12.11	1.00	1034	0.852	3.52	82,817

$z(1) = z(2)/2$ with $z(1) = 0.6z(2)$ necessitates a modification of the maximization problem if equality of human capital within a generation is to be preserved. We therefore replace (65) with

$$2z(1)e_t^0(2\lambda_t, \tau_t(1), 1, \kappa_t)\lambda_t + 1 = z(2)e_t^0(\lambda_t, -g_t(2), 2, \kappa_t)\lambda_t + 1, \quad (70)$$

or, when $z(1) = 0.6z(2)$,

$$1.2e_t^0(2\lambda_t, \tau_t(1), 1, \kappa_t) = e_t^0(\lambda_t, -g_t(2), 2, \kappa_t). \quad (71)$$

Since $e_t \in [0, 1]$, it follows at once from (71) that equality in human capital can be preserved as long as $e_t^0(2\lambda_t, \tau_t(1), 1, \kappa_t) \leq 1/1.2$. Once this condition is violated, the disadvantages of having only one parent, as expressed by $z(1) > z(2)/2$, cannot be overcome by policy interventions.

With this modification of (65), we obtain the series in Tables 8.2 and 8.3 for policy options 2 and 3, respectively. Under policy option 2, the condition $1.2e_t^0(1) = e_t^0(2)$ is maintained until 2080, so that inequality appears first in 2110. Spending on health is still heavier than in section 5.2.4; so, too, are the subsidies to one-parent families and the taxes on those with two parents, since $e_t^0(2) > e_t^0(1)$ must hold for as long as possible. As a result, the growth of λ suffers: in 2080, its level is 17 per cent lower than in section 5.2.4, and only 75 per cent higher than in 1990.

Table 8.2: Variation 2: Policy option 2: nuclear families, lump-sum subsidies

year	λ	$e(1)$	$e(2)$	η	$g(2)$	τ	$\delta(2)$	$\delta(4)$	κ	$y(1)$
1990	3.14	0.46	0.55	1134	3064	5217	1	1	0.855	22,597
2020	3.42	0.53	0.63	1154	3359	5580	1	1	0.855	24,345
2050	4.02	0.67	0.80	1192	4000	6363	1	1	0.856	28,151
2080	5.48	0.99	1.00	1296	5581	8265	1	1	0.856	37,527

Table 8.3: Variation 2: Policy option 3: nuclear families, school-attendance subsidies

year	λ	$e(1)$	$e(2)$	η	$g(2)$	τ	$\delta(2)$	$\delta(4)$	κ	$y(1)$
1990	3.14	0.57	0.68	978	345	1980	1	1	0.850	22,374
2020	3.99	0.80	0.96	1031	354	2250	1	1	0.852	27,698
2050	6.34	1.00	1.00	1031	176	2334	1	1	0.852	43,324

Under policy option 3, however, growth is faster than in section 5.2.5, even though taxation of two-parent families is higher. The reason for this happy result is that the potential advantages stemming from $z(1) > z(2)/2$ are not overwhelmed by the weight of taxes needed to ensure equality when subsidization is so efficiently achieved. In 2020, λ is 1.9 per cent higher than its counterpart in section 5.2.5, and both $e(1)$ and $e(2)$ exceed their (common) counterpart values. In 2050, there is still no inequality in λ among adults, and the common level of 6.34 is now markedly higher than that in section 5.2.5. Thereafter, inequality sets in, as $e_{2050}^0(1) = 1$.

5.3.3 A delayed policy response

In the ‘base-case’, as defined in section 5.2, the outbreak of the disease at the start of period 0 is recognized at once, and an appropriate package of measures to combat it is also implemented without delay. This is one, rather optimistic, extreme possibility. At the other extreme in this 30-year framework is the possibility that nothing at all is, or can be, done to stem the epidemic until the beginning of period 1. In effect, η_0 is constrained to be zero, and

public intervention is limited to promoting education and caring for orphans when the family structure is nuclear.

Such a lag in policy response is arguably fatal under pooling. This can be seen at once by noting from Table 6.2 that the value of $\lambda_1(0)$, namely, 2.01, lies just below the value of $\lambda^*(0, 0.86)$, namely, 2.143. That is to say, if, at the start of period 0, extended families expect the level of $\kappa_0 (= 0.338)$ to prevail also in period 1, they will choose a level of schooling for their children such that $\lambda_1(0)$ lies below even the critical level of λ that corresponds to the disease disappearing for good in period 1, namely, $\lambda^*(0, 0.86)$, let alone the critical level that corresponds to an unabated continuation of the epidemic, namely, $\lambda^*(0, 0.338)$, the value of which is 5.60. This is confirmed by the series in Table 9.1: 2020 sees a slight recovery in e , but the shock is too great and λ continues to decline, despite heavy (optimal) spending on measures to combat the disease, which keeps κ at 0.844. By 2080, full-time child labor rules, with ‘backwardness’ to follow one generation later. This is a sobering illustration of the second part of proposition 2.

Table 9.1: Variation 3, Policy Option 1: Spending on combating the disease under pooling

year	λ	e	η	κ	n	$y(0)$
1960	2.62	0.50	0	0.860	4.65	19,503
1990	3.14	0.19	0	0.338	8.87	26,366
2020	2.00	0.24	853	0.844	3.55	15,563
2050	1.78	0.15	853	0.844	3.55	14,236
2080	1.44	0.00	853	0.844	3.55	12,261

It might be argued that the assumption of stationary expectations in this setting is both implausible and too pessimistic. Despite the shocking level of premature adult mortality in period 0 ($\kappa_0 = 0.338$), the surviving parents might anticipate a successful campaign to contain the disease in period 1 and educate all the children in their care accordingly. To explore this case, we set $\kappa_1 = 0.84$ (a little lower than the level of 0.86 prevailing in the absence of the disease) and allow the system to evolve according to the program set out in earlier sections

thereafter. This change brings about strikingly different results. For all the burden of raising nearly nine children – as opposed to the three or four in section 5.2.3 – each couple chooses a even higher level of education for each one of them – 0.67 as opposed to 0.60. Expectations concerning low mortality in period 1 are more than fulfilled, and growth is even more rapid than when there is no delay.

Table 9.2: Variation 3, Policy Option 1: Forward-looking Expectations

year	λ	e	η	κ	n	$y(0)$
1990	3.14	0.67	0	0.34	8.87	23,498
2020	4.44	0.96	1048	0.85	3.52	30,439
2050	7.95	1.00	1048	0.85	3.52	54,369
2080	14.01	1.00	1048	0.85	3.52	95,773

These results, too, invite skepticism; for the degree of selflessness and altruism implicit in the specification of preferences in (17) when there are almost two adopted children in the ‘family’ for every natural child borders on saintliness. As a further possibility, therefore, let us return to Lemma 4, in which all children must be treated identically, but surviving adults value only the future human capital attained by their natural children. We set $\kappa_1 = 0.84$, and reinstate altruism towards adopted children from 2020 onwards. This yields the following utility specifications:

$$\begin{aligned}
 1990 : \quad EU_t(0) &= 2[u(c_t(0)) + n_t \cdot 0.84 v(2z(0, \kappa_t)f(e_t)\lambda_t + 1)] \\
 2020 - 2080 : \quad EU_t(0) &= 2[u(c_t(0)) + n_t(\kappa_{t+1}/\kappa_t) v(2z(0, \kappa_t)f(e_t)\lambda_t + 1)]
 \end{aligned}$$

The withdrawal of all feelings of altruism towards adopted children under the burden of dependency in period 0, coupled with the social necessity of treating all children alike, leads to an economic collapse, and even more swiftly than when altruism is maintained under stationary expectations, as is seen by comparing Tables 9.1 and 9.3. It is most doubtful that surviving parents would in fact continue to treat their natural children no differently from

Table 9.3: Variation 3, Policy Option 1: Forward-looking Expectations and Limited Altruism towards Adopted Children when Adoption Rates are High

year	λ	e	η	κ	n	$y(0)$
1990	3.14	0.14	0	0.34	8.87	26,718
2020	1.71	0.12	814	0.84	3.57	13,817
2050	1.33	0.00	814	0.84	3.57	11,566
2080	1.00	0.00	814	0.84	3.57	9,277

adopted ones under these grim circumstances. With the breakdown of pooling as a system of complete care for children, it is only to be expected that natural children will be sent to school and adopted children put to work. The ensuing inequalities in the next and later periods will have grave consequences of their own; but they will not be tackled in this paper.

6 Conclusions

The central conclusions of this paper are, first, that the AIDS epidemic will peak far in advance of the economic damage it will ultimately cause. In southern Africa, where prevalence rates among the age-group 15-49 are already 20 per cent and more, the worst is still to come. Second, the scale of that damage, in terms of accumulated losses in GDP per capita, will also be large, even if the measures designed to combat the disease and to ensure the education of half- and full orphans are well chosen and the fiscal means employed to finance them are highly efficient. In the absence of such measures, an economic collapse is on the cards.

The main reason for these gloomy findings lies in the peculiarly insidious and selective character of the disease. By killing mostly young adults, AIDS does more than destroy the human capital embodied in them; it also deprives their children of those very things they need to become economically productive adults – their parents’ loving care, knowledge and capacity to finance education. This weakening of the mechanism through which human

capital is transmitted and accumulated across generations becomes apparent only after a long lag, and it is progressively cumulative in its effects. Therein lies the source of the difference between our findings and those of previous studies, which have focused either on the role of quasi-fixed factors over the medium run or on the historical record to date.

What are the lessons to be drawn for public policy? Where the prevalence rate is still low, as in much of Asia, eastern Europe, the Near East and Latin America, it is of the utmost importance to contain the disease at once: for the economic system as well as for individuals, an ounce of prevention is worth far more than a pound of cure¹⁰.

Where the epidemic is more advanced, combating the disease and its economic effects successfully will require a large and determined fiscal effort, the correct design of which is a complicated matter. The theoretical sections of the paper are devoted to a rigorous formulation and analysis of this problem. Intuitively, the question is: what combination of measures should be adopted to promote the formation of human capital and good health when the threat of a collapse looms? These measures are partly complementary; for the maintenance of good health means that the human capital embodied in individuals during childhood and training will survive and pay off into old age, not only for them, but also for their children. When fiscal resources are very scarce, however, some trade-offs will be unavoidable, requiring the concentration of resources on some programs or groups at the expense of others. The hope here is that our knowledge about what works in the fields of child-rearing, education, the care of orphans, health, and so forth can be distilled into a form where it reveals how to formulate combined programs of interventions that will ward off the threat of an economic collapse. The true social rate of return to such programs can be extremely high, whereas that derived from calculations based on standard ('local') cost-benefit analysis may be quite

¹⁰Sackey and Raparla (2001a, 2001b) use a simple demographic model to compare the costs of prevention (sex education, prevention of mother-to-child transmission, increased condom use and voluntary counselling and testing) with those of mitigation (care of orphans and AIDS patients and payment of pensions) for Namibia and Swaziland. These packages of measures are estimated to cost 2.6 and 6.8 per cent of GDP, respectively, in Namibia, and 1.8 and 4.6 per cent of GDP in Swaziland.

modest. Fiscal policy in general, and policy in the social sectors in particular, must be formulated with a clear eye on its contribution to solving the long-run economic problem posed by AIDS. For in the event of a collapse of productivity, little else will matter.

These points are vividly illustrated by our results for South Africa. In the absence of the epidemic, there would have been the prospect of modest, but accelerating growth of per capita income. As things now stand, the economy could be on the brink of a progressive collapse. With the right interventions, this fate can be averted, though the costs are high, even under favorable social arrangements for the care of orphans. If those arrangements break down, growth is likely to be rather sluggish. We readily concede that these conclusions must be regarded as preliminary, and that various aspects of the calibration process in particular need further work and refinement. Yet the sensitivity analysis already reported above suggests that these findings are robust to changes in a variety of key assumptions and parameter values concerning mortality, the efficacy of measures taken to combat it and the formation of expectations. And it would be unconscionable to err on the side of optimism.

7 Appendices

7.1 An Example

In this section we examine a particular example in order to illustrate the main results of section 2.

7.1.1 Household decisions

We work with the following functional forms:

$$\begin{aligned} f(e_t) &= e_t \\ u(c_t) &= \begin{cases} c_t & \text{if } c_t \geq c^{\min} \\ -\infty & \text{otherwise} \end{cases} \\ v(\lambda_{t+1}) &= \delta \ln(\lambda_{t+1} + \zeta) \text{ with } 0 < \delta < 1 \end{aligned}$$

We further simplify the analysis by assuming that $z(1) = \frac{z(2)}{2} \equiv \bar{z}$.

To examine the household's decisions, we form the Lagrangian for $s_t = 1$ and $s_t = 2$:

$$\begin{aligned} \mathcal{L}(s_t) &= (3 - s_t)c_t + (3 - s_t)n_t\kappa_{t+1}\delta\{\ln(2\bar{z}e_t\lambda_t + 1 + \zeta)\} \\ &\quad + \mu\{\alpha((3 - s_t)\lambda_t + n_t\gamma) - (3 - s_t + n_t\beta)c_t - n_t\alpha\gamma e_t\} \end{aligned}$$

The first-order conditions are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t} &= (3 - s_t) - (3 - s_t + n_t\beta)\mu \leq 0, \quad c_t \geq c^{\min} \quad \text{complementarily} \\ \frac{\partial \mathcal{L}}{\partial e_t} &= \frac{(3 - s_t)n_t\kappa_{t+1}\delta 2\bar{z}\lambda_t}{2\bar{z}e_t\lambda_t + 1 + \zeta} - \mu\alpha n_t\gamma \leq 0, \quad e_t \geq 0 \quad \text{complementarily} \end{aligned}$$

Assuming an interior solution, solving for μ yields

$$\mu = \frac{3 - s_t}{3 - s_t + n_t\beta}$$

Thus, we obtain the optimal choices $e_t^0(s_t)$ as follows:

$$e_t^0(s_t) = \frac{2\kappa_{t+1} \delta \bar{z} \lambda_t (3 - s_t + n_t \beta) - \alpha \gamma (1 + \zeta)}{2\alpha \gamma \bar{z} \lambda_t}, \quad s_t = 1, 2 \quad (72)$$

We immediately observe that $e_t^0(1) > e_t^0(2)$ as long as an interior solution holds for $s_t = 2$. Obviously, $e_t^0(s_t)$ is set to zero if (72) yields a negative number, and it is set to unity if (72) yields a number larger than 1. The budget constraint yields the corresponding interior solution for an adult's consumption:

$$c_t^0(s_t) = \frac{-2\kappa_{t+1} \delta \bar{z} \lambda_t n_t (3 - s_t + n_t \beta) + \alpha \gamma n_t (1 + \zeta) + 2\alpha \lambda_t \bar{z} ((3 - s_t) \lambda_t + n_t \gamma)}{(3 - s_t + n_t \beta) 2\bar{z} \lambda_t} \quad (73)$$

We summarize the properties of the optimal choices in the following proposition.

Proposition 12

Suppose $\zeta > -1$. Then:

- (i) $e_t^0(1) \geq e_t^0(2)$
- (ii) $e^0(s_t) := \lim_{\lambda_t \rightarrow \infty} e_t^0(s_t) = \min \left\{ \frac{\delta \kappa (3 - s_t + n \beta)}{\alpha \gamma}, 1 \right\}$,
where $\lim_{t \rightarrow \infty} \kappa_{t+1} = \kappa$ and $n_t = n \forall t$
- (iii) If $e^0(1) < 1$, then $e^0(1) > e^0(2)$
- (iv) $\frac{\partial e_t^0(s_t)}{\partial \lambda_t} > 0$ for $0 < e_t^0(s_t) < 1$
- (v) $\frac{\partial c_t^0(s_t)}{\partial \lambda_t} > 0$ for all $\lambda_t > \hat{\lambda} \equiv \sqrt{\frac{\gamma(1+\zeta)}{2\bar{z}(3-s_t)}}$

As long as $\lambda_t > \hat{\lambda}$, therefore, the example fulfills all the conditions of the general household model, as set out in section 2.1.1.

7.1.2 Steady states

We now calculate the steady-state value of λ_t associated with positive education levels. For this purpose, we require a stationary mortality profile and stationary fertility: $\kappa_t = \kappa, n_t = n \forall t$.

The steady state is given by:

$$\lambda^*(s_t, \kappa) = 2\bar{z}\lambda^* \frac{(3 - s_t + n\beta) 2\kappa\delta\bar{z}\lambda^* - \alpha\gamma(1 + \zeta)}{2\alpha\gamma\bar{z}\lambda^*} + 1 \quad s_t = 1, 2$$

Solving for $\lambda^*(\cdot)$ yields:

Corollary 2

Suppose $\zeta > 0$ and $\alpha\gamma(\zeta + 1) > (3 - s_t + n\beta)2\delta\kappa\bar{z}$. Then for $s_t = 1, 2$, there exists a unique steady state $\lambda^(s_t, \kappa) > 1$ which is associated with a positive level of education whereby:*

$$\lambda^*(s_t, \kappa) = \frac{\alpha\gamma\zeta}{(3 - s_t + n\beta)2\delta\kappa\bar{z} - \alpha\gamma}$$

Note that $\partial\lambda^*/\partial\kappa < 0$: lower premature mortality among adults is associated with a lower steady-state value of λ . Thus, the steady state has the following properties, which accord with intuition:

Corollary 3

Suppose $\zeta > 0$ and $\alpha\gamma(\zeta + 1) > (3 - s_t + n\beta)2\delta\kappa\bar{z}$. Then,

$$(i) \quad \frac{\partial\lambda^*(s_t, \kappa)}{\partial\kappa} < 0$$

$$(ii) \quad \lambda^*(2, \kappa) > \lambda^*(1, \kappa)$$

$$(iii) \quad \text{there exists a } \hat{\kappa} \in (0, 1) \text{ such that } \lim_{\kappa \rightarrow \hat{\kappa}} \lambda^*(s_t, \hat{\kappa}) = \infty$$

7.2 Proof of Proposition 3

We proceed from period $t = 0$ onwards.

Period 0

There are $[1 - \pi(4)]$ taxpaying households (recall that $N_0 = 1$), each of which pays at least τ^{ba} . The subsidy \bar{g} is paid to beneficiaries, beginning with one-parent households. The proportion of all households receiving this subsidy is therefore

$$\delta_0 = [1 - \pi(4)]\tau^{ba}/\bar{g} < (1 - \pi(4)) \quad (74)$$

where δ_0 is also the proportion of all children born in period 0 who will fully educate their children when they themselves become adults. Since

$$z(1)f[e^0(2, \tau^{ba} - \bar{g}, 1)] \cdot 2 \geq z(2)f[e^0(1, \tau^{ba} - \bar{g}, 2)] \cdot 1, \quad (75)$$

the proportion of adults attaining human capital of at least $\Lambda^a(2) + \tau^{ba}/\alpha$ in period 1 is also δ_0 . Of the rest, $[1 - \pi(4) - \delta_0]n_0$ children are raised by at least one parent, but do not attend school and so attain only $\lambda = 1$ as adults in period 1; and $\pi(4) \cdot n_0$ grow up as full orphans, who cannot, by assumption, pay any taxes as adults in period 1.

Period 1

The children educated in period 0 marry among themselves as adults in period 1. The survivors form a total of $[1 - \pi(4)]\delta_0 N_1$ one- and two-parent families. All can pay τ^{ba} , a tax which would leave them with at least $\alpha\Lambda^a(2)$ in net income and so induce them to choose $e^0 = 1$ for their children.

Since $\delta_0 < [1 - \pi(4)]$, there are also other potential tax-payers. These are the offspring of the one- and two-parent households in period 0 that did not receive a subsidy in that period.¹¹ Of these, the fraction $[1 - \pi(4)]$ will belong to those unions in which at least one parent survives into old age. Hence, the number of tax-payers of this kind is $[1 - \pi(4) - \delta_0][1 - \pi(4)]N_1$.

It follows that the total number of tax-paying households in period 1 is $[1 - \pi(4)]^2 N_1$, each of whom will be able to pay at least τ^{ba} in taxes. Thus, the revenue available for the distribution of subsidies is at least $[1 - \pi(4)]^2 N_1 \cdot \tau^{ba}$, and the number of households that could be paid a subsidy of \bar{g} in period 1 is at least $[1 - \pi(4)]^2 N_1 (\tau^{ba}/\bar{g})$.

¹¹We rule out the payment of subsidies to adults who die prematurely.

The $[1 - \pi(4)]\delta_0 N_1$ households with human capital of at least $\Lambda^a(2) + \tau^{ba}/\alpha$ will choose $e = 1$ in the absence of subsidies. The pool of potential beneficiaries in period 1, excluding full orphans, is $[1 - \pi(4)](1 - \delta_0)N_1$. If $[1 - \pi(4)](1 - \delta_0)N_1 < [1 - \pi(4)]^2 N_1 (\tau^{ba}/g)$, then the task of educating full orphans can begin. Recalling (74) it is seen that all children of one- and two-parent households will attend school full-time in period 1 if $\delta_0 \geq 1/2$.

Analogously to δ_0 , define

$$\delta_1 = \min \left[[1 - \pi(4)]^2 \cdot (\tau^{ba}/\bar{g}), [1 - \pi(4)](1 - \delta_0) \right].$$

Period 2

We begin with the adults attaining at least $z(2)f(1)(\Lambda^a(2) + \tau^{ba}/\alpha) + 1$. The maximal tax that can be imposed on one-parent households of this group without their choosing $e^0 < 1$ is $\alpha [z(2)f(1)(\Lambda^a(2) + \tau^{ba}/\alpha) + 1 - \Lambda^a(2)]^{12}$. The total taxes collected from the whole group amount to at least

$$[1 - \pi(4)]^2 \delta_0 N_2 \cdot \left[\alpha [z(2)f(1)(\Lambda^a(2) + \frac{\tau^{ba}}{\alpha}) + 1 - \Lambda^a(2)] \right] > [1 - \pi(4)]^2 \delta_0 N_2 \cdot \tau^{ba} \quad (76)$$

by virtue of $z(2)f(1) > 1$.

The group attaining $\Lambda^a(2) + \tau^{ba}/\alpha$ is analyzed as in period 1. This group pays taxes in the amount $[1 - \pi(4)]\delta_1 N_2 \cdot \tau^{ba}$.

Period t

In order to see whether this process eventually generates sufficient funds to promote everyone, including full orphans, we concentrate on the group that was originally promoted through subsidies in period 0. The total taxes paid by this (heterogeneous) group in period t exceed

$$\Psi_t := [1 - \pi(4)]^t \cdot \alpha [\Lambda_t(2) - \Lambda^a(2)] \cdot \delta_0 N_t,$$

where $\Lambda_t(2)$ is the human capital of the adult in a single-parent family that is the last in an unbroken sequence of such households from period 0 onwards. The capacity of these

¹²Note that a higher tax can be imposed on the two-parent households.

revenues to finance subsidies in a growing population with $\pi(4) > 0$ requires normalization by N_t . We have

$$\begin{aligned}\lim_{t \rightarrow \infty} \Psi_t / N_t &= \lim_{t \rightarrow \infty} \alpha \cdot \delta_0 [1 - \pi(4)]^t \cdot \Lambda_t(2) \\ &= \alpha \cdot \delta_0 \lim_{t \rightarrow \infty} [1 - \pi(4)]^t \cdot [z(2)f(1)[z(2)f(1)[\dots]]\Lambda^a(2) + 1]\end{aligned}$$

Hence, $\lim_{t \rightarrow \infty} \Psi_t / N_t = \infty$ if and only if $(1 - \pi(4)) \cdot z(2)f(1) > 1$. The same holds for all other groups that are ‘promoted’ subsequently. Assumption 12 then yields the desired result.

7.3 Proof of proposition 5

We proceed by comparing double and single targeting.

Double Targeting

In period $t = 0$ with $N_0 = 1$, let each of some group of families receive simultaneously health and education support of $2\bar{h}$ and \bar{g} , respectively, so that all the adults will survive to old age. The share of all families that can be supported in this way is given by:

$$\delta_0 = \frac{B_0}{\bar{g} + 2\bar{h}} \tag{77}$$

In $t = 0$, therefore, we obtain the following societal pattern:

- $\delta_0 n_0$ children reach, as adults, $\lambda_1 = z(1)f(e^0(2, -\bar{g}, 1)) \cdot 2 + 1$
- $(1 - p)^2(1 - \delta_0)n_0$ children are left as full orphans, who reach $\lambda_1 = \xi$
- $(2p - p^2)(1 - \delta_0)n_0$ children go uneducated, and reach $\lambda_1 = 1$

In period $t = 1$, the educated children marry among themselves. All such families can pay $\bar{\tau}$ in taxes, provided they receive continued health support in the amount of \bar{h} per adult. With

the revenues $B_1 = \bar{\tau} \delta_0 n_0 / 2$, the assumption that $\bar{\tau} > 2\bar{h}$ implies that the society can afford to subsidize a further share of families, denoted by δ_1 , on the same pattern as in period 0 by:¹³

$$\frac{n_0}{2} \bar{\tau} \delta_0 = \{\delta_0 2\bar{h} + \delta_1 (\bar{g} + 2\bar{h})\} \frac{n_0}{2}$$

which yields

$$\delta_1 = \frac{\delta_0 (\bar{\tau} - 2\bar{h})}{\bar{g} + 2\bar{h}} \quad (78)$$

To sum up,

$$\frac{n_0}{2} (\delta_0 + \delta_1) = \frac{n_0}{2} \frac{B_0}{\bar{g} + 2\bar{h}} \left\{ 1 + \frac{\bar{\tau} - 2\bar{h}}{\bar{g} + 2\bar{h}} \right\} = \frac{n_0}{2} \frac{B_0}{\bar{g} + 2\bar{h}} \frac{\bar{g} + \bar{\tau}}{\bar{g} + 2\bar{h}} \quad (79)$$

families whose children will attain at least $z(1)f(e^0(2, -\bar{g}, 1)) \cdot 2 + 1$ in period 2. For period 2 onwards, we assume that the society operates under an optimal education and health support system whose precise nature we do not (need to) specify further.

Single Targeting

In period $t = 0$, each of some group of families receives the transfer \bar{g} . The share of families so supported is given by:

$$w_0 = \frac{B_0}{\bar{g}}. \quad (80)$$

This process generates the following structure:

- $p^2 w_0 n_0$ children reach $\lambda_1 = z(1)f(e^0(2\alpha + \bar{g}, 1)) \cdot 2 + 1$
- $2p(1 - p)w_0 n_0$ children reach $\lambda_1 = \Lambda^a(2) + \frac{\tau^{ba}}{\alpha}$
- $(1 - p)^2 w_0 n_0$ children are left as full orphans, who reach $\lambda_1 = \xi$
- another $(1 - p)^2(1 - w_0)n_0$ children become orphans, reaching $\lambda_1 = \xi$
- $(2p - p^2)(1 - w_0)n_0$ children have at least one parent, but receive no education, and so reach $\lambda_1 = 1$

¹³We assume here that the adults subsidized in $t = 1$ were not orphans in $t = 0$.

In period $t = 1$, the adults who were educated in period $t = 0$ can be taxed as long as they do not die prematurely¹⁴. Hence:

$$B_1 = \left[p^2 \bar{\tau} + 2p(1-p)\tau^{ba} \right] \frac{w_0 n_0}{2} \quad (81)$$

Therefore, the society may be able to subsidize a further share of w_1 households,¹⁵ even while those educated in period 0 will be fully protected against premature mortality in period 1. The budget constraint is $B_1 \geq (2p - p^2)\bar{h}w_0 n_0 + (\bar{g} + 2\bar{h})w_1 \frac{n_0}{2}$, which yields

$$w_1 = w_0 \frac{2p(\tau^{ba} - 2\bar{h}) - p^2(2\tau^{ba} - 2\bar{h} - \bar{\tau})}{\bar{g} + 2\bar{h}}. \quad (82)$$

Under *ST*, therefore, the society supports $[(2p - p^2)w_0 + w_1] \frac{n_0}{2}$ families over two periods in such a way that their offspring reach at least $\Lambda^a(2) + \tau^{ba}/\alpha$ in period 2. Note that the share of families newly subsidized in $t = 1$ receive support in both domains and are therefore double targeted. This makes both systems comparable.¹⁶ From $t = 2$ onwards, the society operates under the same optimal program as under *DT*.

Comparison

We begin by noting that $z(1)f(e^0(2, -\bar{g}, 1)) \cdot 2 + 1 \geq z(2)f(e^0(1, -\bar{g}, 2)) \cdot 1 + 1$. In order to compare *DT* and *ST*, it suffices to compare $(\delta_0 + \delta_1) \frac{n_0}{2}$ with $[(2p - p^2)w_0 + w_1] \frac{n_0}{2}$, since under both systems the offspring of these families will be fully educated and able to pay taxes when they enter period $t = 2$ as adults. Therefore, *DT* is superior to *ST* if $\delta_0 + \delta_1 > (2p - p^2)w_0 + w_1$. We observe from (77) and (80) that

$$\delta_0/w_0 = \bar{g}/(\bar{g} + 2\bar{h}).$$

Hence, from (79) and (82) and some algebraic manipulations, it follows that *DT* is superior to *ST* if

¹⁴Note that we assume here that only educated adults are taxed.

¹⁵Again, we assume that the adults subsidized in $t = 1$ were not orphans in $t = 0$.

¹⁶Essentially, we compare a sequence of education and health subsidies with simultaneous subsidies over one generation.

$$\frac{\bar{g}}{\bar{g}+2\bar{h}} \left\{ 1 + \frac{\bar{\tau}-2\bar{h}}{\bar{g}+2\bar{h}} \right\} > (2p-p^2) \left\{ \left[1 + \frac{\tau^{ba}-2\bar{h}}{\bar{g}+2\bar{h}} \right] + \frac{p^2}{2p-p^2} \left[\frac{\bar{\tau}-\tau^{ba}}{\bar{g}+2\bar{h}} \right] \right\}$$

Now the expression in brackets on the LHS of this inequality is greater than that in braces on the RHS for all $p \in (0, 1)$ if $\bar{\tau} > \tau^{ba}$; they are equal if $\bar{\tau} = \tau^{ba}$. It follows that DT is superior to ST if $\frac{\bar{g}}{\bar{g}+2\bar{h}} > (2p-p^2)$ or, equivalently,

$$\frac{\bar{g}}{\bar{h}} > \frac{4p-2p^2}{(1-p)^2}$$

7.4 Proof of Proposition 6

We go through the same line of reasoning employed in the proof of proposition 5. Under DT with health spending $\bar{\eta}$, there will be no premature adult mortality, and the share of all families that can be given a transfer in the amount \bar{g} in period 0 is

$$\delta_0 = \frac{B_0 - \bar{\eta}}{\bar{g}},$$

where $B_0 \geq \tau^{ba}$ and, by hypothesis, $B_0 \geq \bar{\eta}$. Total revenue in period 1 is then

$$B_1 = [\delta_0 \bar{\tau} + (1 - \delta_0) \tau^{ba}] N_1,$$

so that the share of all families that can be given a transfer in the amount \bar{g} in period 1 is

$$\delta_1 = [\delta_0 \bar{\tau} + (1 - \delta_0) \tau^{ba} - \bar{\eta}] / \bar{g} \geq (\tau^{ba} - \bar{\eta}) / \bar{g},$$

where the weak inequality holds as an equality if and only if $\tau^{ba} = \bar{\tau}$.

Under ST, we have the state vector $[\pi(1), \pi(2), \pi(3), \pi(4)]$ in period 0. The share of all families that can be given a transfer in the amount \bar{g} in period 0 is

$$w_0 = B_0 / \bar{g} = \delta_0 + (\bar{\eta} / \bar{g}).$$

This policy yields the following structure of human capital levels in period 1:

- $\pi(1)w_0n_0$ children attain $z(1)f(e^0(2\alpha + \bar{g}, 1)) \cdot 2 + 1$ and can pay $\frac{\bar{\tau}}{2}$ in $t + 1$;
- $[\pi(2) + \pi(3)]w_0n_0$ children attain $\Lambda^a(2) + \tau^{ba}$;
- $[1 - \pi(4)](1 - w_0)n_0$ have been raised by at least one parent and attain unity;
- $\pi(4)n_0$ are left as full orphans, and attain ξ .

Note that $\pi(1) = p^2$, $\pi(2) = \pi(3) = p(1 - p)$ and $\pi(4) = (1 - p)^2$. Suppose $\bar{\eta}N_1$ is affordable. Then the gross tax revenue in period 1 will be¹⁷

$$[\pi(1)w_0\bar{\tau} + [(\pi(2) + \pi(3))w_0 + (1 - \pi(4))(1 - w_0)]\tau^{ba}]N_1,$$

so that $\bar{\eta}N_1$ is indeed affordable if and only if

$$\bar{\eta} \leq \pi(1)w_0\bar{\tau} + [(\pi(2) + \pi(3))w_0 + (1 - \pi(4))(1 - w_0)]\tau^{ba}.$$

Since $w_0 = B_0/\bar{g}$, some manipulation yields the equivalent condition

$$B_0 \geq \frac{[\bar{\eta} - (1 - \pi(4))\tau^{ba}] \cdot \bar{g}}{\pi(1)(\bar{\tau} - \tau^{ba})},$$

which is always satisfied if $\bar{\eta} \leq (1 - \pi(4))\tau^{ba}$.

If this condition is violated, however, what we have called ST is, in fact, infeasible.

Suppose, therefore, that the first condition in the proposition does hold. Analogously to δ_0 , we have

$$w_1 = [\pi(1)w_0\bar{\tau} + [(\pi(2) + \pi(3))w_0 + (1 - \pi(4))(1 - w_0)]\tau^{ba} - \bar{\eta}]/\bar{g}.$$

Now recall that

$$z(1)f(e^0(2, -\bar{g}, 1)) \cdot 2 + 1 \geq z(2)f(e^0(1, -\bar{g}, 1)) + 1.$$

¹⁷Note that we assume in this proposition that all adults can be taxed.

Hence, in order to establish the superiority of DT over ST, it suffices to compare $(\delta_0 + \delta_1) \frac{n_0}{2}$ with $[(1 - \pi(4))w_0 + w_1] \frac{n_0}{2}$; for under both schemes, the offspring of these families will be fully educated and able to pay taxes when they enter period 2 as adults. Combining the expressions derived above, we have

$$\delta_0 + \delta_1 > (1 - \pi(4))w_0 + w_1$$

if and only if

$$\delta_0 \left[1 + \frac{\bar{\tau} - \tau^{ba}}{\bar{g}} \right] + \frac{\tau^{ba} - \bar{\eta}}{\bar{g}} > w_0 \left[(1 - \pi(4)) + \pi(1) \frac{\bar{\tau} - \tau^{ba}}{\bar{g}} \right] + \frac{(1 - \pi(4))\tau^{ba} - \bar{\eta}}{\bar{g}}.$$

Some tedious manipulation reveals that this condition can be written as

$$B_0 > \frac{\bar{\eta}[\bar{g} + (\bar{\tau} - \tau^{ba})] - \pi(4)\tau^{ba}\bar{g}}{\pi(4)\bar{g} + (1 - \pi(1))(\bar{\tau} - \tau^{ba})}, \quad (83)$$

which establishes the result.

7.5 Proof of proposition 7

- (i) If parents in period 0 expect the epidemic to continue in period 1, then by assumption 15, no child will attain $\lambda^*(1, 1)$ in period 1, regardless of what actually happens in period 1. Given the educational decisions in period 0, the government will be able to collect $\Theta_1(\kappa_1 < 1)N_1$ in taxes. If $\Theta_1 < \bar{\eta}$, there will be insufficient revenue to suppress the epidemic in period 1, thereby fulfilling parents's expectations in period 0 that the epidemic will be rampant. The fact that no child attains $\lambda^*(1, 1)$ in period 1 implies that $\Theta_2(\kappa_2 < 1) < \Theta_1(\kappa_1 < 1)$, so that the epidemic continues in period 2, and so forth. The result is a progressive collapse.

If, however, $\Theta_1(\kappa_1 < 1) \geq \bar{\eta}$, the funds to suppress the epidemic will be available, even with the limited investment in education in period 0 that would occur if parents expected the epidemic to continue in period 1.

- (ii) Suppose, next, that parents in period 0 expect the epidemic to be suppressed in period 1 and choose e_0 accordingly. Hence, the children of one- and two-parent families will

reach $\lambda_1(2, 1)$ and $\lambda_1(1, 1)$, respectively, as adults in period 1. Now the maximum that a family can pay in taxes in period 1 without its children failing to reach $\lambda^*(1, 1)$ in period 2 is $\alpha[\Lambda_1(s_0, \kappa = 1) - \Lambda^*(s_0, 1)]$. Hence, if the government is to be able to suppress the epidemic in period 1 without destroying the basis for growth in subsequent periods, it must be the case that

$$\sum_{s_0=1}^3 \alpha \pi(s_0) \cdot [\Lambda_1(s_0, 1) - \Lambda^*(s_0, 1)] N_1 > \bar{\eta} N_1,$$

or

$$\alpha[2\pi(1) \cdot \lambda_1(1, 1) + (\pi(2) + \pi(3))\lambda_1(2, 1) - 2\pi(1) \cdot \lambda^*(1, 1) - (\pi(2) + \pi(3))\lambda^*(2, 1)] > \bar{\eta}.$$

Suppose this condition is satisfied. Then the expectations at time $t = 0$ will be fulfilled and the total human capital of all lineages stemming from $s_0 = 1, 2, 3$ will be greater in period 2 than in period 1, thereby permitting the continued suppression of the epidemic while also providing some revenues to subsidize, and hence 'promote', the lineages stemming from $s_0 = 4$. Since the former lineages will eventually experience growth in human capital at the rate $2z(1)f(1) - 1 > 0$, the whole society will eventually enter the condition of self-sustaining growth, with the epidemic fully suppressed in all periods.

If, however, the said condition is violated, the government will be able to finance $\bar{\eta} N_1$ in period 1 only by so depriving at least some of the lineages stemming from $s_0 = 1, 2, 3$ that their children fail to attain at least $\lambda^*(1, 1)$ in period 2. The ensuing destruction of the tax base in period 2 may be so large as to make suppression of the epidemic in that, or some subsequent, period infeasible.

References

- [1] Arndt, C. and Lewis, J. (2000), 'The Macro Implications of HIV/AIDS in South Africa: A Preliminary Assessment', *South African Journal of Economics*, 68(5): 856-87
- [2] Barro, R. and Lee, J.W., (1993), 'International Data on Schooling Years and Schooling Quality', *American Economic Review, Papers and Proceedings*, 86(2): 218-23.
- [3] Barro, R., and Sala-I-Martin, X. (1995), *Economic Growth*, New York: McGraw-Hill.
- [4] Basu, K. (1999), 'Child Labor: Cause, Consequence, and Cure, with Remarks on International Labor Standards', *Journal of Economic Literature*, 38, 1083-1119.
- [5] Basu, K. and P.H. Van (1998), 'The Economics of Child Labor', *American Economic Review*, 88, 412-427.
- [6] Bell, C., and Gersbach, H. (2002), 'Child Labor and the Education of a Society', Alfred Weber Institute, University of Heidelberg.
- [7] Bloom, D. and Canning, D. (2000), 'Health and the Wealth of Nations', *Science*, 287: 1207-9.
- [8] Bloom, D., Canning, D., and Sevilla, J. (2001), 'The Effect of Health on Economic Growth: Theory and Evidence', *NBER Working Paper*, No. 8587.
- [9] Bonnel, R. (2000), 'HIV/AIDS: Does it Increase or Decrease Growth in Africa?', ACT, Africa Department, Washington, DC, World Bank.
- [10] Case, S.C., Paxson, C.H., and Ableidinger, J.D. (2002), 'Orphans in Africa', *NBER Working Paper*, No. 9213.
- [11] Kambou, G., Devarajan, S., and Over. M. (1992), 'The Economic Impact of AIDS in an African Country; Simulations with a CGE Model of Cameroon', *Journal of African Economies* 1: 109-30.

- [12] Dorrington, R., *et al.* (2001), 'The Impact of HIV/AIDS on Adult Mortality in South Africa', Technical Report, Medical Research Council, Tygerburg, South Africa.
- [13] Marseille, E., Hofmann, P.B., and Kahn, J.G. (2002), 'HIV Prevention before HAART in Sub-Saharan Africa, *The Lancet*, 359: 1851-6.
- [14] Over, A.M. (1992), 'The Macroeconomic Impact of AIDS in Sub-Saharan Africa', AFTPN Technical Working Paper 3, Population, Health and Nutrition Division, Africa Technical Department, Washington, DC, World Bank.
- [15] Sackey, J., and Raparla, T., (2000), 'Lesotho: The Development, Impact of HIV/AIDS - Selected Issues and Options', World Bank Report No. 21103-LSO
- [16] Sackey, J., and Raparla, T., (2001a), 'Swaziland: Selected Development Impact of HIV/AIDS', World Bank Report No. 22044-SW.
- [17] Sackey, J., and Raparla, T., (2001ab), 'Namibia: Selected Development Impact of HIV/AIDS', World Bank Report No. 22046-NA.
- [18] U.N.AIDS (2002), *Report on the Global HIV/AIDS Epidemic, 2002*, New York.
- [19] W.H.O. (2001), 'Macroeconomics and Health: Investing in Health for Economic Development', Geneva.
- [20] Wolff, P.H. (2002), 'Eritrea: Collective Responsibility for War Orphans', *IK Notes*, No. 50, World Bank.
- [21] World Bank (2002), *World Development Indicators*, Washington, DC.

8 Figures

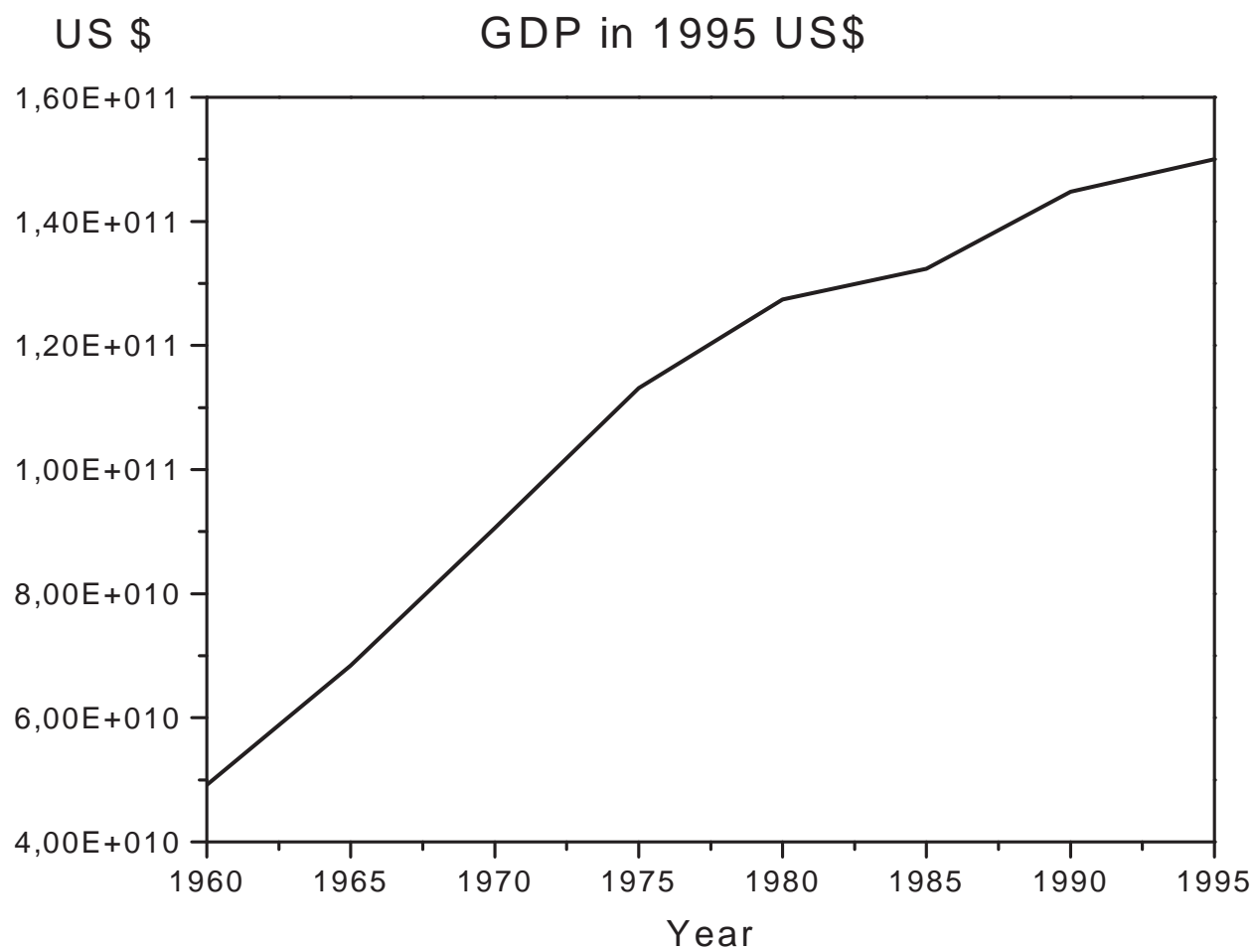


Figure 2: GDP in 1995 US\$

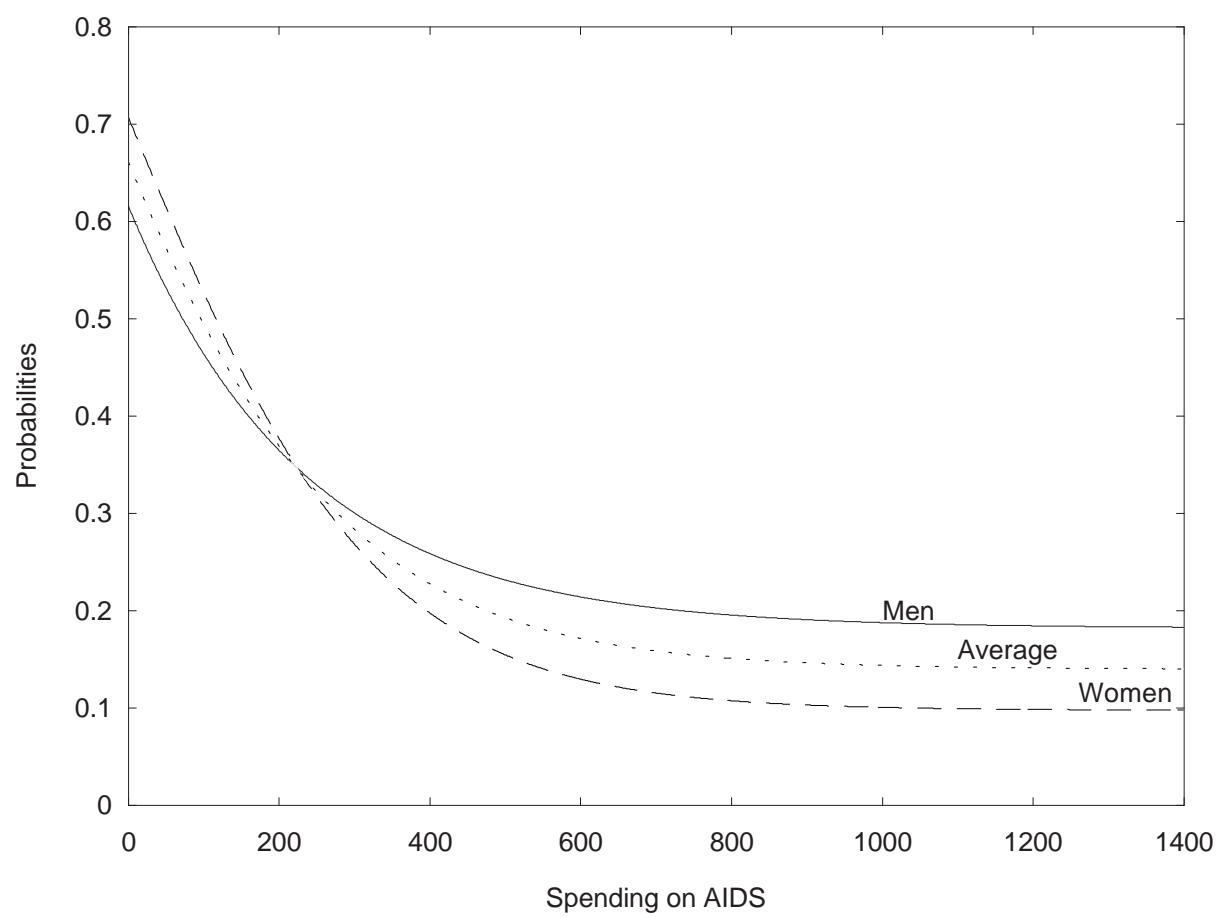


Figure 3: Mortality ${}_{30}q_{20}$ as a function of η

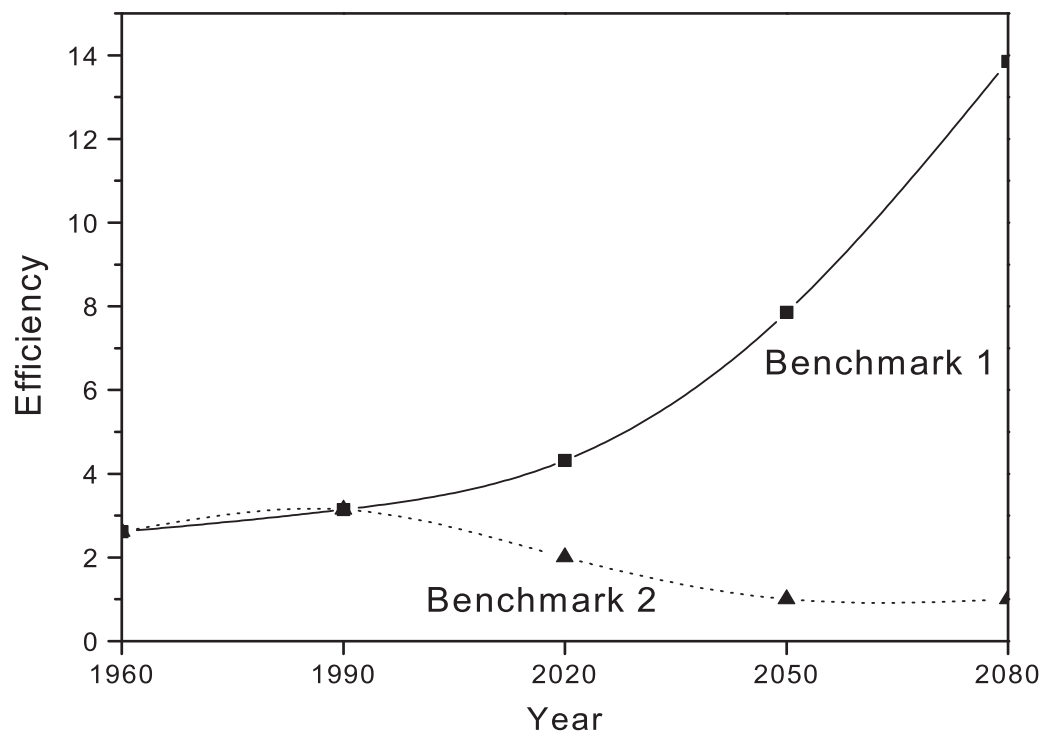


Figure 4: Benchmarks 1 and 2

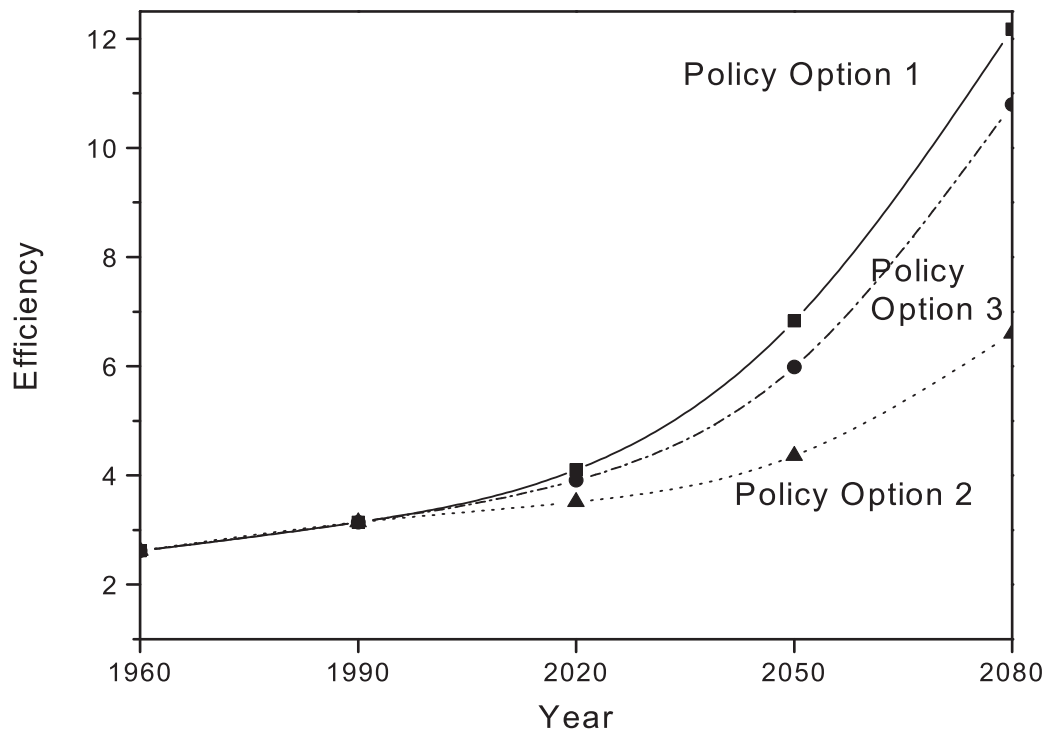


Figure 5: Policy options 1,2 and 3

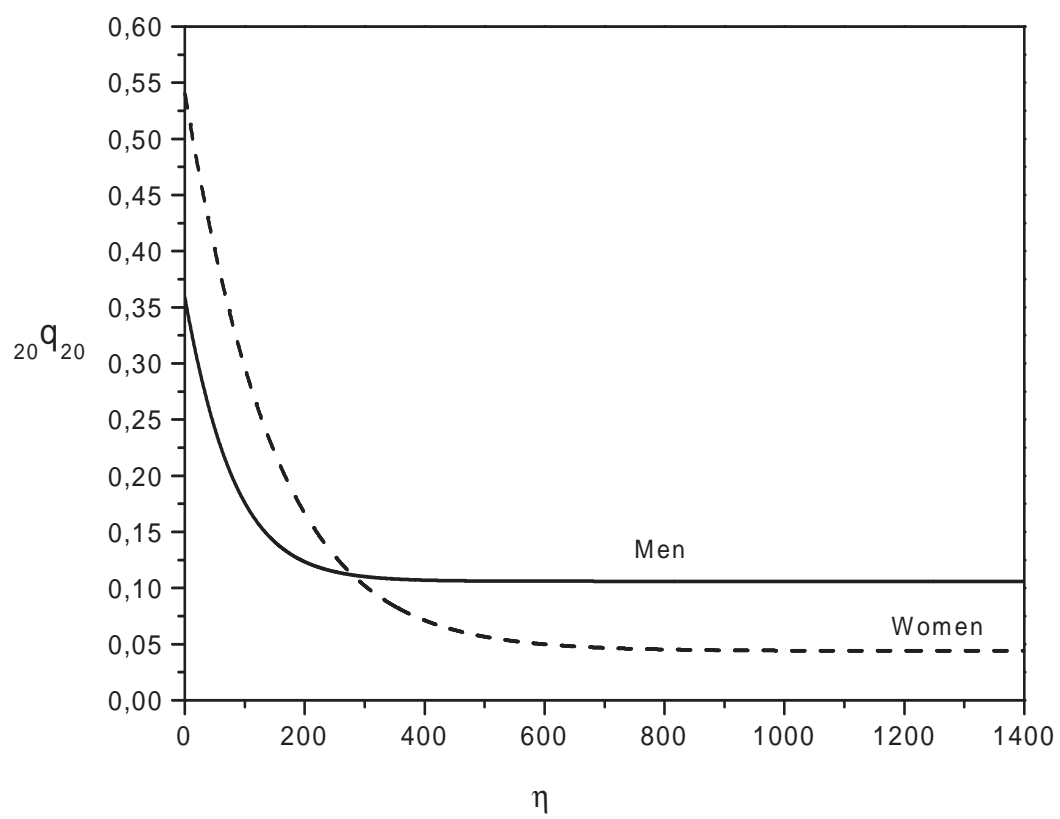


Figure 6: Mortality (${}_{20}q_{20}$) as a function of η

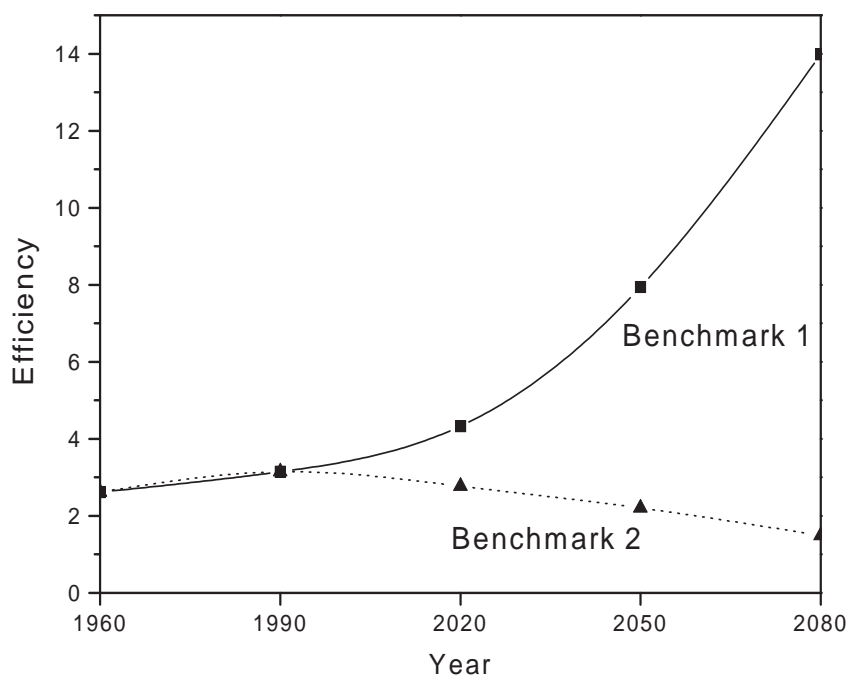


Figure 7: Variation 1: The benchmarks under $20q_{20}$

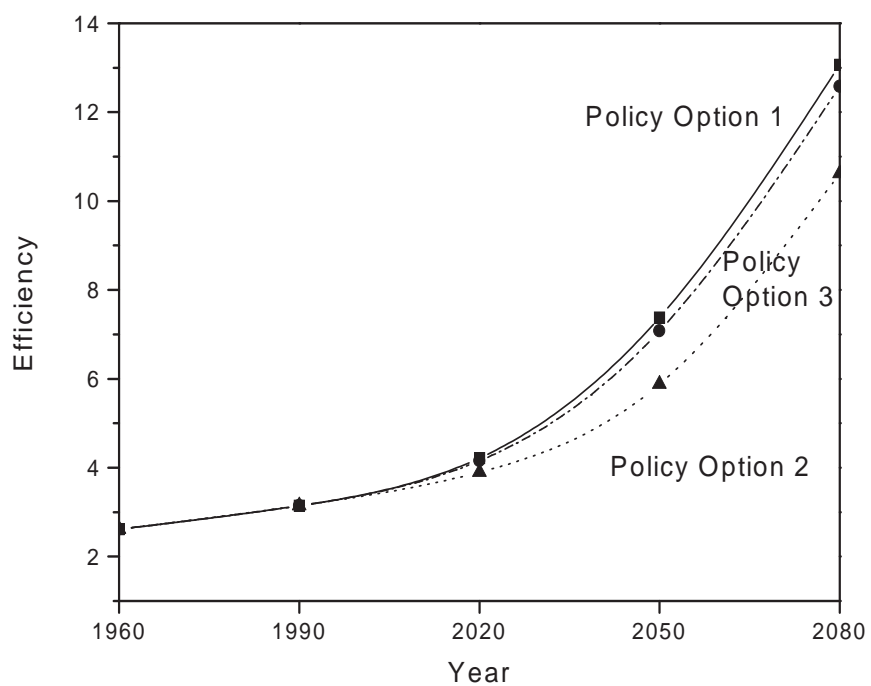


Figure 8: Policy Options 1, 2 and 3 under $20q_{20}$

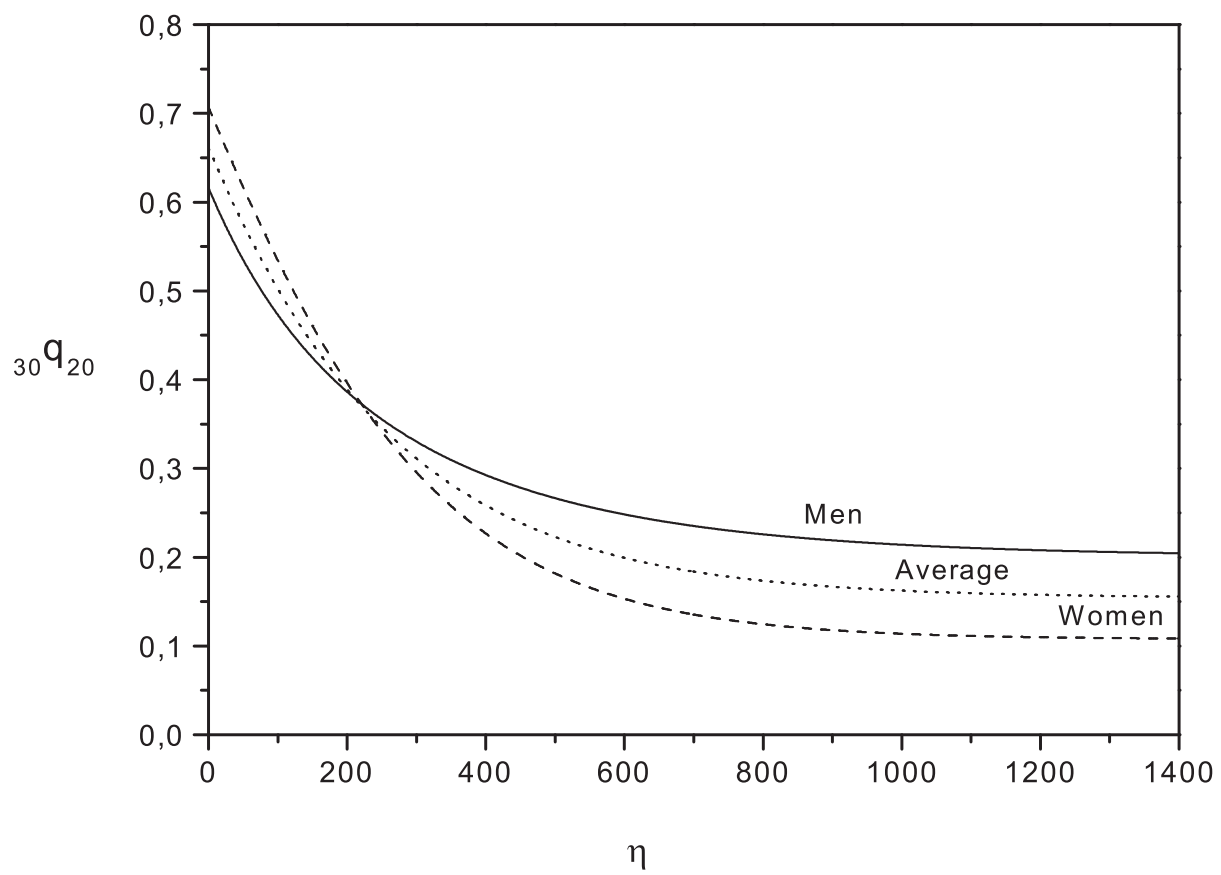


Figure 9: Variation 1: A permanent shift in mortality (${}_{30}q_{20}$)

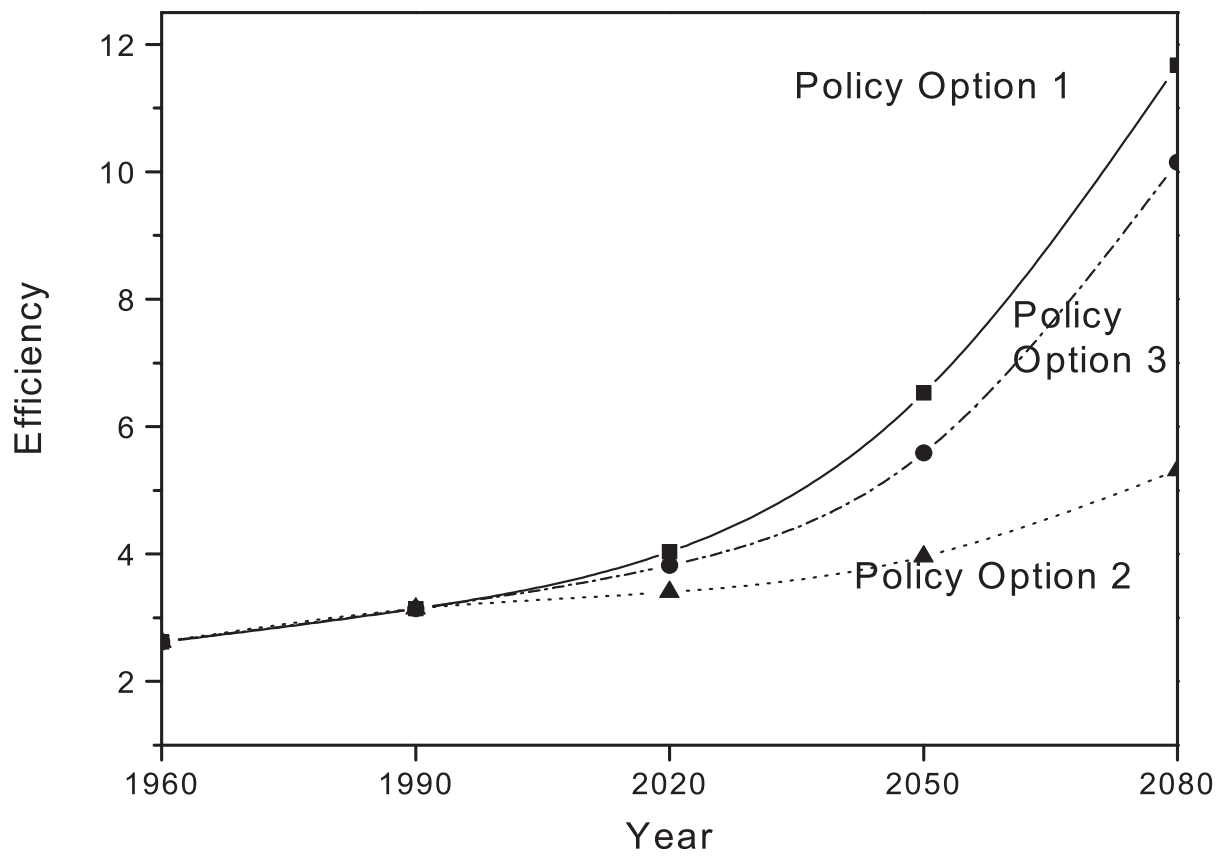


Figure 10: Variation 1: Policy options 1,2 and 3 ($_{30}q_{20}$)

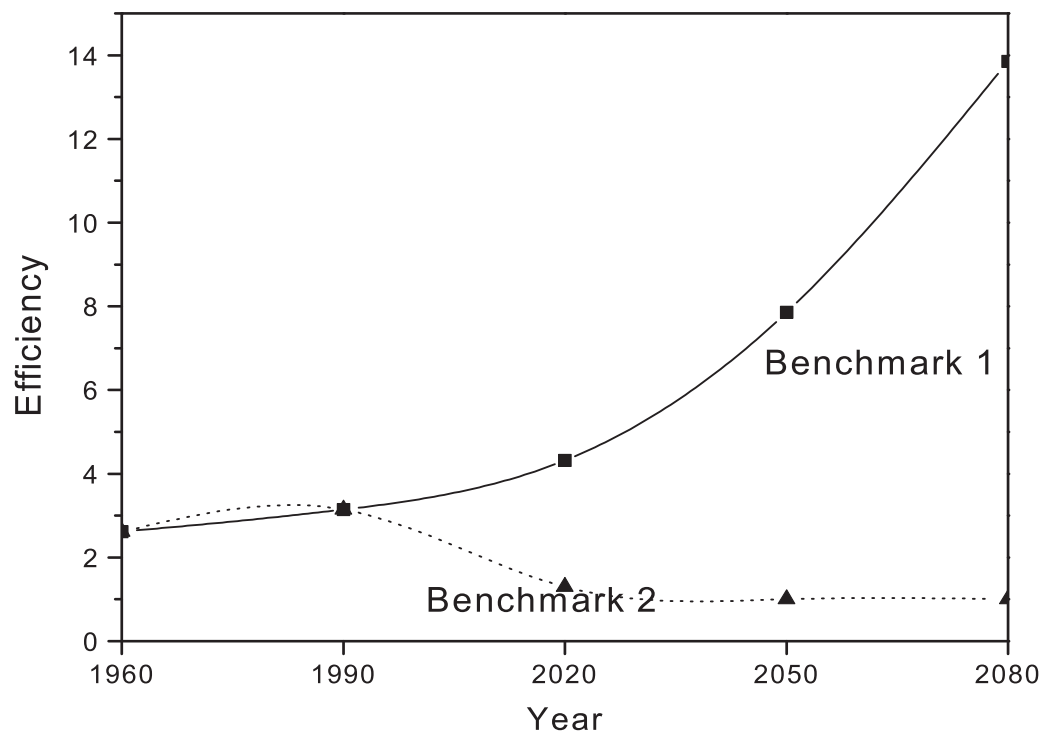


Figure 11: Variation 2: Benchmarks 1 and 2

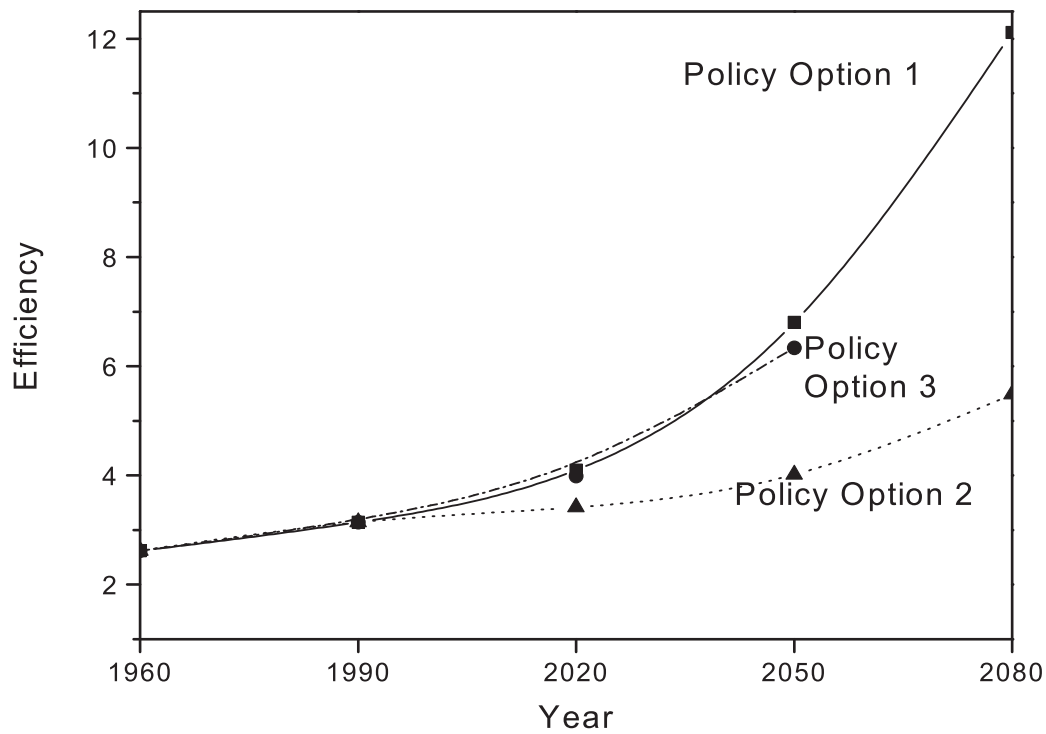


Figure 12: Variation 2: Policy options 1,2 and 3